Gravitational Wave Background search with Pulsar Timing Arrays

Tutorial given at the IPTA Student Week 2026 by Aurélien Chalumeau

Big thanks to Daniel Reardon, Serena Valtolina and Valentina DiMarco for their help !

Table of Content:

In [1]:
```
# Use this to allow tab-completion
%config Completer.use_jedi = False
%config IPCompleter.omit__names = 0

from __future__ import division

print('\nLoading modules ...', end='')

# System
import shutil, os, glob, sys
import multiprocessing as mp

# Maths
import numpy as np
import numpy.ma as ma
import scipy.linalg as sl
from scipy import stats
from scipy.integrate import quad
from scipy.stats import norm
from scipy.optimize import curve_fit
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, WhiteKernel

# Plots
import matplotlib
```

```
import matplotlib.pyplot as plt
from matplotlib.lines import Line2D
from matplotlib.patches import Patch
import matplotlib.colors as mcolors
from matplotlib.colors import LogNorm
import corner

# Astro general
from astropy.coordinates import SkyCoord
import astropy.units as u

# PTA specifics

## libstempo
import libstempo as LT

## enterprise & enterprise_extensions
import enterprise
from enterprise.pulsar import Pulsar
from enterprise.signals import parameter, gp_priors, gp_signals, white_
from enterprise.signals.gp_bases import createfourierdesignmatrix_dm
from enterprise_extensions.sampler import JumpProposal

from enterprise_extensions.blocks import white_noise_block, red_noise_b
from enterprise_extensions import hypermodel
from enterprise_extensions.sampler import save_runtime_info
from enterprise_extensions.frequentist import optimal_statistic as osta

# la_forge
from la_forge.core import Core

# defiant
import defiant
from defiant import OptimalStatistic
from defiant import utils, orf_functions
from defiant import plotting as defplot
from defiant.null_distribution import phase_shift_OS, sky_scramble_OS

# PTMCMC sampler
from PTMCMCSampler.PTMCMCSampler import PTSampler as ptmcmc

print("OK !")
```

```
Loading modules ...
/opt/build_psrsoft/workspace/micromamba/envs/IPTA_Env/lib/python3.11/si
te-packages/astropy/config/paths.py:55: AstropyUserWarning: XDG_CONFIG_
HOME is set to '/home/pulsar/.config', but the default location, /home/
pulsar/.astropy/config, already exists, and takes precedence. This envi
ronment variable will be ignored.
  return set_temp_config._get_dir_path(rootname)
/opt/build_psrsoft/workspace/micromamba/envs/IPTA_Env/lib/python3.11/si
te-packages/enterprise/signals/utils.py:13: UserWarning: pkg_resources
is deprecated as an API. See https://setuptools.pypa.io/en/latest/pkg_r
esources.html. The pkg_resources package is slated for removal as early
as 2025-11-30. Refrain from using this package or pin to Setuptools<81.
  from pkg_resources import Requirement, resource_filename
OK !
```

In [2]:

```
# Matplotlib settings
matplotlib.rcParams['mathtext.fontset'] = 'stix'
```

```
matplotlib.rcParams['font.family'] = 'STIXGeneral'
linestyles = "-"
linewidths = 1.0
shade = True
shade_gradient = 2.0
shade_alpha = 0.2
colors = mcolors.CSS4_COLORS['mediumblue']
sigmas = [1, 2]
tfs = 18 #tick
lfs = 27 #label
```

1. Introduction: The Gravitational Wave Background in the context of Pulsar Timing Arrays

Useful functions

In [3]:
```
def get_HD_curve(zeta):
    coszeta = np.cos(zeta * np.pi / 180)
    x = np.where(zeta == 0, 1., (1 - coszeta) / 2)  # avoid log(0)
    HD = 3/2 * (1/3 + x * (np.log(x) - 1/6))
    return np.where(zeta == 0, 1., HD)
```

1.1 What is the Gravitational Wave Background (GWB)?

A **GWB** is a stochastic superposition of gravitational wave signals from a large number of unresolved sources. Pulsar Timing Arrays (PTAs) are sensitive to the GWB in the **nanohertz band** (10^{-9} – 10^{-6} Hz), where the main expected sources are:

1. The population of **supermassive black hole binaries** (SMBHBs)
2. Topological defects:
   A. **Cosmic strings**: 1D topological defects from phase transitions in the early Universe
   B. **Domain walls**: 2D topological defects from discrete symmetry breaking
3. **Cosmological first order phase transitions**, PTAs

are sensitive at the QCD scale

4. Primordial GWBs:
   A. **Inflationary GWs**. from the amplification of quantum vacuum fluctuations of the gravitational field during inflation
   B. **Scalar-induced GWs**, second-order effect, arising from enhanced primordial scalar perturbations at second order in perturbation theory

> PTA data sets might contain all (or a sub-set) of these **astrophysical** (for the SMBHB) and **cosmological** (for the others) signals, thus separating and constraining them will likely become one of the main challenges.

1.2 More about the GW spectrum from SMBHBs

The GW signal from a population of SMBHBs is made of a **stochastic background** (the incoherent superposition of unresolved binaries) and **bright single sources** (individually resolvable binaries above the background).

The figure below shows a simulated GW spectrum (orange line) made of thousands of single sources (small black points), while few of them could rise over the global population at their emitted GW frequency (**thick black points**). The **black solid line** displays a power-law that might fit well with such a GWB, and the red dotted line shows a typical PTA sensitivity level.

GWB spectrum

1.3. Standard approximations

The GWB searched by PTAs is often modeled under three standard assumptions:

Assumption	Meaning
Isotropic	The signal power is uniformly distributed across the sky
Stationary	Statistical properties do not evolve over the observation time
Unpolarized	No preferred polarization state; equal power in $+$ and $\times$ modes

We will follow these assumptions in this tutorial.

Key consequence 1:

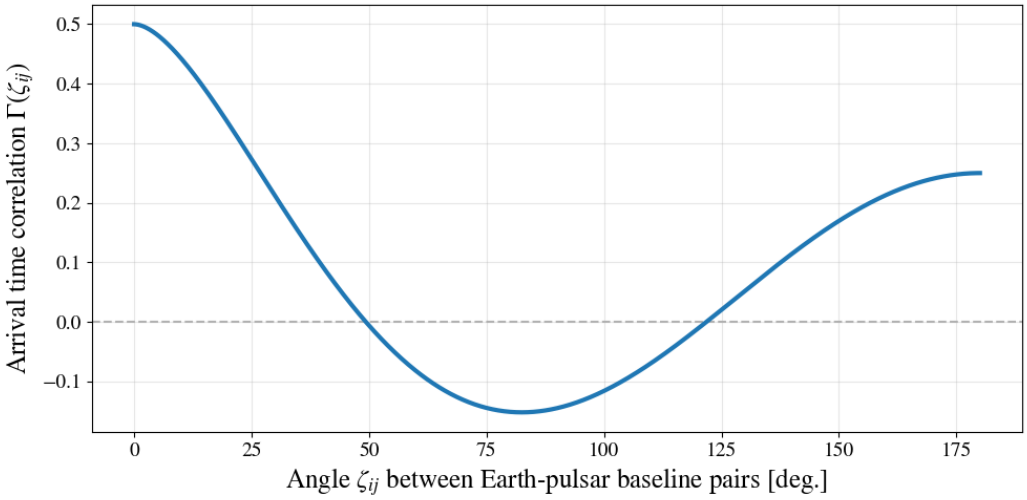
Under these three approximations, the spatial correlations between any two pulsars i and j depend **only on their angular separation** ζ_{ij} and is defined by the famous **Hellings-Downs relation** $\Gamma(\zeta_{ij})$, which describes the level of correlations within the pulses arrival times between two pulsars, as function of their angular separation, as

$$\Gamma(\zeta_{ij}) = \frac{3}{2}\left[\frac{1}{3} + x_{ij}\left(\ln x_{ij} - \frac{1}{6}\right)\right] + \frac{1}{2}\delta_{ij}, \quad x_{ij} = \frac{1-\cos\zeta_{ij}}{2}.$$

Here, we use a normalisation so that $\Gamma(0) = 0.5$.

Here is how it actually looks like

In [4]:
```
zeta = np.linspace(1e-5,180,1000)
HD = get_HD_curve(zeta)

plt.figure(figsize=(10,5))
plt.plot(zeta, HD, lw=3)
plt.axhline(0, c='k', ls='--', zorder=0, alpha=.3)
plt.xlabel("Angle $\zeta_{ij}$ between Earth-pulsar baseline pai
plt.ylabel("Arrival time correlation $\Gamma(\zeta_{ij})$", font
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)
plt.tight_layout()
plt.grid(alpha=.3)
plt.show()
```



The Hellings-Downs (HD) correlation is the **smoking gun** of the GWB: detecting it in PTA data is what distinguishes a true GW signal from any other noise or signal.

Key consequence 2:

The GWB is fully characterized by its **one-sided power spectral density**: $S_h(f)$

We typically model a power-law and mention the predicted gammas

$$S_h(f) = \frac{A^2}{12\pi^2}\left(\frac{f}{f_{\rm ref}}\right)^{-\gamma} f^{-3} \quad [{\rm s}^3]$$

where A is the GWB characteristic strain amplitude at $f_{\rm ref}$, γ the spectral index, which is equal to $13/3$ for a population of circular and GW-driven SMBHBs, and $f_{\rm ref}$ is typically $1\,{\rm yr}^{-1}$.

With this tutorial, we will

- Vizualize a simulated PTA data set made of 133 pulsars
- See how we could model the GWB signal in PTA data sets
- See how to run a **Bayesian analysis** to (1) constrain the GWB spectral properties.
- See how to run the **Optimal Statistic** approach (OS) to (1) recover the GWB amplitude, (2) recover the HD curve and (3) compute the "frequentist" significance of the signal
- A Bonus part will follow right after that

Some selected references related to Section 1

- The Nanohertz Gravitational Wave Astronomer (Taylor, 2021), very useful book about PTA.
- Upper limits on the isotropic gravitational radiation background from pulsar timing analysis (Hellings & Downs, 1983), reference for the Hellings & Downs correlations
- A Practical Theorem on Gravitational Wave Backgrounds (Phinney, 2001), reference for the predicted power-law slope for a GW-driven and circular population of supermassive black hole binaries

2. The simulated IPTA data sets (2 versions here)

Useful functions

In [5]:
```
#############
### Plot Pulsar positions in a sky map projection
#############

def plot_SkyMap(psrs, coordtype, psrcolor='w', show_mw=True)
    # Initiate and set Figure
    fig = plt.figure(figsize=(12, 6))
    fig.patch.set_facecolor('k')
    ax = fig.add_subplot(111, projection="mollweide") # proj
    ax.set_facecolor('k')

    # Milky Way disk
    if show_mw:
        Plot_MW(ax, coordtype=coordtype, n_stars=5e5, angle_

    # Pulsars
    for psr in psrs:
        if coordtype=="gal":
            coord = SkyCoord(ra=psr._raj * u.rad, dec=psr._de
            gal = coord.galactic
            x = gal.l.rad
            if x > np.pi:
                x -= 2 * np.pi
            y = gal.b.rad
        elif coordtype=="eq":
            x  = psr._raj - np.pi   # shift [0,2π] → [-π,π]
            y = psr._decj
        ax.plot(x, y, marker='*', markerfacecolor=psrcolor, 

    # Set Legends
    legend_elements = [
        Line2D([0], [0], marker='*', color=psrcolor, label='
        Patch(facecolor='darkorange', edgecolor='k', lw=1, l
    leg = plt.legend(handles=legend_elements, fontsize=16, l

    # Set Axes
    xticks = np.linspace(-np.pi, np.pi, 13)
    if coordtype=="gal":
        xtick_labels = [f"{int(lon)}°" for lon in np.linspac
        ax.set_xlabel('Galactic Longitude $l$', fontsize=20,
        ax.set_ylabel('Galactic Latitude $b$', fontsize=20, 
    elif coordtype=="eq":
        xtick_labels = [f"{int(h)}h" for h in np.linspace(0,
        ax.set_xlabel('RA [hr]', fontsize=18, color='white')
        ax.set_ylabel('DEC [deg]', fontsize=18, color='white
    ax.set_xticks(xticks)
    ax.set_xticklabels(xtick_labels, fontsize=20, color='whi
    ax.tick_params(colors='white', labelsize=20)

    # Set Mollweide frame/grid
```

```
    ax.grid(alpha=0.4, color='white', zorder=1)
    for spine in ax.spines.values():
        spine.set_edgecolor('white')

    plt.tight_layout()
    plt.show()

def Plot_MW(ax, coordtype="eq", n_stars=500000, angle_mask=4
    rng = np.random.default_rng(42)
    b_thin  = rng.normal(0, np.deg2rad(3),  int(n_stars * 0.8
    b_thick = rng.normal(0, np.deg2rad(15), int(n_stars * 0.1
    b_halo  = rng.normal(0, np.deg2rad(40), int(n_stars * 0.0
    b_fake  = np.clip(np.concatenate([b_thin, b_thick, b_halo

    # --- Longitude with galactic-center concentration ---
    # Sample l from a mixture: a narrow Gaussian at l=0 (bulg
    # + a broad Gaussian (disk) + a uniform floor (halo/backg
    n = len(b_fake)
    l_bulge   = rng.normal(0, np.deg2rad(15),  int(n * 0.25))
    l_disk    = rng.normal(0, np.deg2rad(100),  int(n * 0.55)
    l_uniform= rng.uniform(-np.pi, np.pi,     int(n * 0.20))
    l_fake    = np.concatenate([l_bulge, l_disk, l_uniform])
    # Wrap to [-π, π]
    l_fake    = (l_fake + np.pi) % (2 * np.pi) - np.pi
    l_fake    = l_fake[:n]

    n_bins_x, n_bins_y = 720, 360

    if coordtype == "eq":
#         l_fake = rng.uniform(0, 2*np.pi, int(n_stars))[:len

        coords   = SkyCoord(l=l_fake*u.rad, b=b_fake*u.rad, f
        x_fake   = np.clip(coords.ra.rad - np.pi, -np.pi, np.p
        y_fake   = coords.dec.rad

        h, xe, ye = np.histogram2d(x_fake, y_fake, bins=[n_bi
                                   range=[[-np.pi, np.pi], [-
        xc = 0.5 * (xe[:-1] + xe[1:])
        yc = 0.5 * (ye[:-1] + ye[1:])
        X, Y = np.meshgrid(xc, yc)

    elif coordtype == "gal":
#         l_fake = rng.uniform(-np.pi, np.pi, int(n_stars))[

        h, xe, ye = np.histogram2d(l_fake, b_fake, bins=[n_bi
                                   range=[[-np.pi, np.pi], [-
        xc = 0.5 * (xe[:-1] + xe[1:])
        yc = 0.5 * (ye[:-1] + ye[1:])
        X, Y = np.meshgrid(xc, yc)

    # Same normalization logic for both branches
    h_masked = np.ma.masked_where(h < angle_mask, h)
    vmax = np.percentile(h[h > 0], 99.5)

    ax.pcolormesh(X, Y, h_masked.T,
                  norm=LogNorm(vmin=1, vmax=vmax),
                  cmap='inferno', shading='auto',
                  zorder=0, rasterized=True, alpha=0.6)

############
```

```
### Plot HD correlations
#############

def get_zetas(psrs):
    zetas = []
    for i, psr1 in enumerate(psrs):
        for j, psr2 in enumerate(psrs):
            if j <= i:
                continue
            angle = np.arccos(np.clip(np.dot(psr1.pos, psr2.
            zetas.append(angle)
    return np.array(zetas)

def plot_HD_from_psrlist(psrs, bin_size_deg):
    # Get pulsar pair angles
    zetas = get_zetas(psrs)

    plt.figure(figsize=(8,5))
    ax = plt.gca()
    plt.suptitle(f"{len(psrs)} pulsars - {len(zetas)} pairs"

    HD = get_HD_curve(zetas)
    ax.plot(zetas, HD, '.', c='k', ms=5, alpha=1)
    ax.axhline(0, c='k', zorder=0, alpha=.4)
    ax.axvline(0, c='k', ls='--', alpha=.3)
    ax.axvline(180, c='k', ls='--', alpha=.3)

    ax.set_xlabel("Angle $\zeta_{ij}$ between Earth-pulsar ba
    ax.set_ylabel("Arrival time correlation $\Gamma(\zeta_{i
    ax.tick_params(axis='both', which='major', labelsize=14)
    ax.grid(alpha=.3, zorder=0)

    ax2 = ax.twinx()
    bin_edges   = np.arange(0, 180 + bin_size_deg, bin_size_
    bin_centers = bin_edges[:-1] + bin_size_deg / 2
    bin_counts, _ = np.histogram(zetas, bins=bin_edges)
    ax2.bar(bin_centers, bin_counts, edgecolor='black', widt

    ax2.set_ylabel('Histogram: number of pairs', fontsize=16
    ax2.set_ylim(0, 2*np.max(bin_counts))
    ax2.tick_params(axis='both', which='major', labelsize=14

    plt.tight_layout()
    plt.show()

#############
### Plot timing residuals
#############

def mjd2greg(mjd):
    return 2000 + (np.array(mjd)-51544.5)/365.25

def greg2mjd(greg):
    return (np.array(greg) - 2000) * 365.25 + 51544.5

def plot_pulsar_timing(psr, prefit_res=None, plot_histo=False

    # --- Layout logic ---
    nrows = 2 if prefit_res is not None else 1
    ncols = 2 if plot_histo else 1
```

```
fig, axes = plt.subplots(
    nrows, ncols,
    figsize=(15 if plot_histo else 13, 5 if nrows == 1 el
    gridspec_kw={'width_ratios': [6, 1] if plot_histo els
                 'hspace': 0., 'wspace': 0.}
)

# Normalize axes array shape
if nrows == 1:
    axes = np.array([axes])
if ncols == 1:
    axes = axes.reshape(nrows, 1)

if prefit_res is None:
    y = 1.05
else:
    y = 1.09


fontsize = 14

fig.suptitle(f"{psr.name}{addtitle}", fontsize=fontsize+1

# --- Color handling ---
if color is None:
    color = psr.freqs
    colorname = "Frequency [$MHz$]"

cmap = mcolors.LinearSegmentedColormap.from_list(
    'coral_blue', ['coral', 'cornflowerblue']
)
norm = mcolors.Normalize(vmin=np.min(color), vmax=np.max



def plot_residual_panel(ax, residuals):
    for toa, res, err, cval in zip(psr.toas, residuals, p
        ax.errorbar(
            toa/86400,
            res*1e6,
            yerr=err*1e6,
            fmt='.',
            ms=6,
            capsize=2,
            color=cmap(norm(cval))
        )

    ax.axhline(0., ls=':', c='k', lw=2)
    ax.grid(alpha=0.2)

# Prefit (top)
if prefit_res is not None:
    ax_prefit = axes[0, 0]
    plot_residual_panel(ax_prefit, prefit_res)
    ax_prefit.set_ylabel('Prefit residuals\n[$\mu s$]', 
    ax_prefit.tick_params(axis='both', which='major', lal
    plt.setp(ax_prefit.get_xticklabels(), visible=False)

    # Upper x-axis with Gregorian dates
```

```
        ax_top = ax_prefit.twiny()
        ax_top.set_xlim(ax_prefit.get_xlim())
        # Pick ~6 evenly spaced tick positions in MJD
        display_cadence = 2 # years
        greg_labels = [int(i) for i in np.arange(int(mjd2gre
        upperticks_mjd = greg2mjd(greg_labels)
        ax_top.set_xticks(upperticks_mjd)
        ax_top.set_xticklabels(greg_labels, fontsize=fontsiz
        ax_top.set_xlabel('Date', fontsize=fontsize)
        ymin, ymax = ax_top.get_ylim()
        ylim = max(abs(ymin), abs(ymax))
        ax_top.set_ylim(-ylim, ylim)
        ax_prefit.set_xticks(upperticks_mjd, minor=True)
        ax_prefit.grid(which='minor', axis='x', alpha=1, lw=

    # Postfit (bottom or only)
    ax_post = axes[-1, 0]
    plot_residual_panel(ax_post, psr.residuals)
    ax_post.set_xlabel('Epoch [MJD]', fontsize=fontsize)
    ax_post.set_ylabel('Timing residuals\n[$\mu s$]', fontsi
    ax_post.tick_params(axis='both', which='major', labelsiz
    ymin, ymax = ax_post.get_ylim()
    ylim = max(abs(ymin), abs(ymax))
    ax_post.set_ylim(-ylim, ylim)

    if prefit_res is None:
        # Upper x-axis with Gregorian dates
        ax_top = ax_post.twiny()
        ax_top.set_xlim(ax_post.get_xlim())
        # Pick ~6 evenly spaced tick positions in MJD
        display_cadence = 2 # years
        greg_labels = [int(i) for i in np.arange(int(mjd2gre
        upperticks_mjd = greg2mjd(greg_labels)
        ax_top.set_xticks(upperticks_mjd)
        ax_top.set_xticklabels(greg_labels, fontsize=fontsiz
        ax_top.set_xlabel('Date', fontsize=fontsize)

    ax_post.set_xticks(upperticks_mjd, minor=True)
    ax_post.grid(which='minor', axis='x', alpha=1, lw=1, ls=

    # --- Colorbar ---
    cax = ax_post.inset_axes([0.02, 0.01, 0.4, 0.03])
    sm = plt.cm.ScalarMappable(cmap=cmap, norm=norm)
    sm.set_array([])
    cb = fig.colorbar(sm, cax=cax, orientation='horizontal')
    cax.xaxis.set_label_position('top')
    cax.xaxis.tick_top()

    if colorname is None:
        cb.set_label('Color scale', size=fontsize, labelpad=
    else:
        cb.set_label('Frequency [MHz]', size=fontsize, label
    cb.ax.tick_params(labelsize=fontsize)


    # --- Histogram(s) ---
    if plot_histo:
        def plot_hist(ax, residuals):
            wr = residuals / psr.toaerrs
            counts, bins = np.histogram(wr, bins=30)
```

```
            counts = counts / counts.max()
            ax.stairs(counts, bins, orientation="horizontal"

            # Gaussian reference
            g = np.random.randn(10000)
            c2, b2 = np.histogram(g, bins=30)
            c2 = c2 / c2.max()
            ax.stairs(c2, b2, orientation="horizontal", lw=2
            ax.legend(fontsize=fontsize)

            ax.grid(alpha=0.4)
            plt.setp(ax.get_xticklabels(), visible=False)
            plt.setp(ax.get_yticklabels(), visible=False)
            ymin, ymax = ax.get_ylim()
            ylim = max(abs(ymin), abs(ymax))
            ax.set_ylim(-ylim, ylim)
            ax.yaxis.set_label_position("right")
            ax.yaxis.tick_right()
            ax.set_ylabel("Weighted residuals", fontsize=fon
            # Remove y ticks AND tick labels
            ax.set_yticks([])
            ax.tick_params(axis='y', which='both', length=0)

        if prefit_res is not None:
            plot_hist(axes[0, 1], prefit_res)

        plot_hist(axes[-1, 1], psr.residuals)

    plt.show()
```

2.1 Main properties

We work with a simulated IPTA data set, for which par/tim files are located here

```
./DataSim/ideal/PTAdata/pulsarname/
pulsarname.par
./DataSim/ideal/PTAdata/pulsarname/
pulsarname.tim
```

Dataset: "ideal"

Main properties of the data set

- **Pulsars:** all the same isolated pulsars, but positions placed at real PTA pulsar positions
- **Epoch properties:** 30 days cadence from

MJD 55000 to MJD 61000, thus spanning 6000 days (~16 years),
- **Errorbars** randomly sampled from 0.05 μs to 1 μs
- **3 observing radio frequencies** among 400, 1200 and 1400 MHz, not important here as we do not consider chromatic (i.e., radio-frequency dependent) noise

What the timing residuals contain

- **White noise:** `EFAC` only
- **GWB**: An Isotropic, unpolarized, stationary and Gaussian Common Red Signal with Hellings-Downs spatial correlations, with amplitude $A = 10^{-15}$ and $\gamma = 13/3 \approx 4.33$ (predicted from a GW-driven population of circular SMBHBs).

Let us finally start to play with the notebook, by reading and vizualizing the chosen data set.

2.2 Reading the parfiles & timfiles with libstempo (a python wrapper for Tempo2)

In [6]:
```
currentdir = os.getcwd()
print('Current directory: '+currentdir)
```

```
Current directory: /home/pulsar/Downloads/GWB-search
```

In [7]:
```
# Choose the data set
dataset = "ideal_for_IPTASW"

# Define data dir
datadir = f'{currentdir}/DataSim/{dataset}/'

# Find all par/tim directories
partimdirs = np.sort(glob.glob(f"{datadir}/PTAdata/*"))

# Define pulsar list
```

```
psrnames = [os.path.basename(d).split("/")[0] for d in par

# Define lists of parfiles and timfiles
parfiles = [f"{d}/{psrnames[i]}.par" for i,d in enumerate(
timfiles = [f"{d}/{psrnames[i]}.tim" for i,d in enumerate(
```

In [8]:
```
# Read the data with libstempo and fit for the timing mode
psrs = []
prefit_res = {}
for parfile, timfile in zip(parfiles, timfiles):
    ltpsr = LT.tempopulsar(parfile, timfile)
    prefit_res.update({ltpsr.name:np.copy(ltpsr.residuals(
    ltpsr.fit()
    psr = Pulsar(ltpsr)
    psrs.append(psr)
    print(ltpsr.name)
```

```
[tempo2Util.C:396] Warning: [TIM1] Please place MODE flags
in the parameter file
WARNING: Could not find pulsar distance for PSR J0023+092
3. Setting value to 1 with 20% uncertainty.
J0023+0923
[tempo2Util.C:401] Warning: [DUP1] duplicated warnings hav
e been suppressed.
J0030+0451
WARNING: Could not find pulsar distance for PSR J0034-053
4. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J0101-642
2. Setting value to 1 with 20% uncertainty.
J0034-0534
J0101-6422
WARNING: Could not find pulsar distance for PSR J0125-232
7. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J0154+183
3. Setting value to 1 with 20% uncertainty.
J0125-2327
WARNING: Could not find pulsar distance for PSR J0218+423
2. Setting value to 1 with 20% uncertainty.
J0154+1833
WARNING: Could not find pulsar distance for PSR J0340+413
0. Setting value to 1 with 20% uncertainty.
J0218+4232
WARNING: Could not find pulsar distance for PSR J0406+303
9. Setting value to 1 with 20% uncertainty.
J0340+4130
J0406+3039
J0437-4715
WARNING: Could not find pulsar distance for PSR J0509+085
6. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J0557+155
1. Setting value to 1 with 20% uncertainty.
J0509+0856
J0557+1551
WARNING: Could not find pulsar distance for PSR J0605+375
7. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J0610-210
0. Setting value to 1 with 20% uncertainty.
```

```
J0605+3757
J0610-2100
WARNING: Could not find pulsar distance for PSR J0614-332
9. Setting value to 1 with 20% uncertainty.
J0613-0200
WARNING: Could not find pulsar distance for PSR J0621+100
2. Setting value to 1 with 20% uncertainty.
J0614-3329
WARNING: Could not find pulsar distance for PSR J0636+512
8. Setting value to 1 with 20% uncertainty.
J0621+1002
WARNING: Could not find pulsar distance for PSR J0636-304
4. Setting value to 1 with 20% uncertainty.
J0636+5128
WARNING: Could not find pulsar distance for PSR J0645+515
8. Setting value to 1 with 20% uncertainty.
J0636-3044
WARNING: Could not find pulsar distance for PSR J0709+045
8. Setting value to 1 with 20% uncertainty.
J0645+5158
J0709+0458
WARNING: Could not find pulsar distance for PSR J0732+231
4. Setting value to 1 with 20% uncertainty.
J0711-6830
WARNING: Could not find pulsar distance for PSR J0740+662
0. Setting value to 1 with 20% uncertainty.
J0732+2314
WARNING: Could not find pulsar distance for PSR J0751+180
7. Setting value to 1 with 20% uncertainty.
J0740+6620
WARNING: Could not find pulsar distance for PSR J0824+002
8. Setting value to 1 with 20% uncertainty.
J0751+1807
WARNING: Could not find pulsar distance for PSR J0900-314
4. Setting value to 1 with 20% uncertainty.
J0824+0028
WARNING: Could not find pulsar distance for PSR J0931-190
2. Setting value to 1 with 20% uncertainty.
J0900-3144
WARNING: Could not find pulsar distance for PSR J0955-615
0. Setting value to 1 with 20% uncertainty.
J0931-1902
J0955-6150
WARNING: Could not find pulsar distance for PSR J1012-423
5. Setting value to 1 with 20% uncertainty.
J1012+5307
WARNING: Could not find pulsar distance for PSR J1017-715
6. Setting value to 1 with 20% uncertainty.
J1012-4235
J1017-7156
J1022+1001
J1024-0719
WARNING: Could not find pulsar distance for PSR J1036-831
7. Setting value to 1 with 20% uncertainty.
J1036-8317
```

```
WARNING: Could not find pulsar distance for PSR J1101-642
4. Setting value to 1 with 20% uncertainty.
J1045-4509
WARNING: Could not find pulsar distance for PSR J1125+781
9. Setting value to 1 with 20% uncertainty.
J1101-6424
WARNING: Could not find pulsar distance for PSR J1125-582
5. Setting value to 1 with 20% uncertainty.
J1125+7819
WARNING: Could not find pulsar distance for PSR J1125-601
4. Setting value to 1 with 20% uncertainty.
J1125-5825
WARNING: Could not find pulsar distance for PSR J1216-641
0. Setting value to 1 with 20% uncertainty.
J1125-6014
WARNING: Could not find pulsar distance for PSR J1231-141
1. Setting value to 1 with 20% uncertainty.
J1216-6410
WARNING: Could not find pulsar distance for PSR J1312+005
1. Setting value to 1 with 20% uncertainty.
J1231-1411
WARNING: Could not find pulsar distance for PSR J1327-075
5. Setting value to 1 with 20% uncertainty.
J1312+0051
WARNING: Could not find pulsar distance for PSR J1421-440
9. Setting value to 1 with 20% uncertainty.
J1327-0755
WARNING: Could not find pulsar distance for PSR J1431-574
0. Setting value to 1 with 20% uncertainty.
J1421-4409
WARNING: Could not find pulsar distance for PSR J1435-610
0. Setting value to 1 with 20% uncertainty.
J1431-5740
WARNING: Could not find pulsar distance for PSR J1446-470
1. Setting value to 1 with 20% uncertainty.
J1435-6100
WARNING: Could not find pulsar distance for PSR J1453+190
2. Setting value to 1 with 20% uncertainty.
J1446-4701
J1453+1902
J1455-3330
WARNING: Could not find pulsar distance for PSR J1513-255
0. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1514-494
6. Setting value to 1 with 20% uncertainty.
J1513-2550
J1514-4946
WARNING: Could not find pulsar distance for PSR J1518+490
4. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1525-554
5. Setting value to 1 with 20% uncertainty.
J1518+4904
WARNING: Could not find pulsar distance for PSR J1543-514
9. Setting value to 1 with 20% uncertainty.
J1525-5545
```

```
WARNING: Could not find pulsar distance for PSR J1545-455
0. Setting value to 1 with 20% uncertainty.
J1543-5149
J1545-4550
WARNING: Could not find pulsar distance for PSR J1547-570
9. Setting value to 1 with 20% uncertainty.
J1547-5709
J1600-3053
J1603-7202
WARNING: Could not find pulsar distance for PSR J1614-223
0. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1629-690
2. Setting value to 1 with 20% uncertainty.
J1614-2230
WARNING: Could not find pulsar distance for PSR J1630+373
4. Setting value to 1 with 20% uncertainty.
J1629-6902
J1630+3734
J1640+2224
J1643-1224
WARNING: Could not find pulsar distance for PSR J1652-483
8. Setting value to 1 with 20% uncertainty.
J1652-4838
WARNING: Could not find pulsar distance for PSR J1653-205
4. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1658-532
4. Setting value to 1 with 20% uncertainty.
J1653-2054
J1658-5324
WARNING: Could not find pulsar distance for PSR J1705-190
3. Setting value to 1 with 20% uncertainty.
J1705-1903
WARNING: Could not find pulsar distance for PSR J1708-350
6. Setting value to 1 with 20% uncertainty.
J1708-3506
WARNING: Could not find pulsar distance for PSR J1710+492
3. Setting value to 1 with 20% uncertainty.
J1710+4923
J1713+0747
WARNING: Could not find pulsar distance for PSR J1719-143
8. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1721-245
7. Setting value to 1 with 20% uncertainty.
J1719-1438
J1721-2457
J1730-2304
WARNING: Could not find pulsar distance for PSR J1737-081
1. Setting value to 1 with 20% uncertainty.
J1732-5049
J1737-0811
WARNING: Could not find pulsar distance for PSR J1738+033
3. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1741+135
1. Setting value to 1 with 20% uncertainty.
```

```
J1738+0333
J1741+1351
J1744-1134
WARNING: Could not find pulsar distance for PSR J1745+101
7. Setting value to 1 with 20% uncertainty.
J1745+1017
WARNING: Could not find pulsar distance for PSR J1745-095
2. Setting value to 1 with 20% uncertainty.
J1745-0952
WARNING: Could not find pulsar distance for PSR J1747-403
6. Setting value to 1 with 20% uncertainty.
J1747-4036
WARNING: Could not find pulsar distance for PSR J1751-285
7. Setting value to 1 with 20% uncertainty.
J1751-2857
WARNING: Could not find pulsar distance for PSR J1757-532
2. Setting value to 1 with 20% uncertainty.
J1757-5322
WARNING: Could not find pulsar distance for PSR J1801-141
7. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1802-212
4. Setting value to 1 with 20% uncertainty.
J1801-1417
WARNING: Could not find pulsar distance for PSR J1804-271
7. Setting value to 1 with 20% uncertainty.
J1802-2124
J1804-2717
WARNING: Could not find pulsar distance for PSR J1804-285
8. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1811-240
5. Setting value to 1 with 20% uncertainty.
J1804-2858
WARNING: Could not find pulsar distance for PSR J1825-031
9. Setting value to 1 with 20% uncertainty.
J1811-2405
WARNING: Could not find pulsar distance for PSR J1832-083
6. Setting value to 1 with 20% uncertainty.
J1825-0319
WARNING: Could not find pulsar distance for PSR J1843-111
3. Setting value to 1 with 20% uncertainty.
J1832-0836
WARNING: Could not find pulsar distance for PSR J1843-144
8. Setting value to 1 with 20% uncertainty.
J1843-1113
J1843-1448
WARNING: Could not find pulsar distance for PSR J1853+130
3. Setting value to 1 with 20% uncertainty.
J1853+1303
J1857+0943
WARNING: Could not find pulsar distance for PSR J1902-510
5. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1903+032
7. Setting value to 1 with 20% uncertainty.
J1902-5105
```

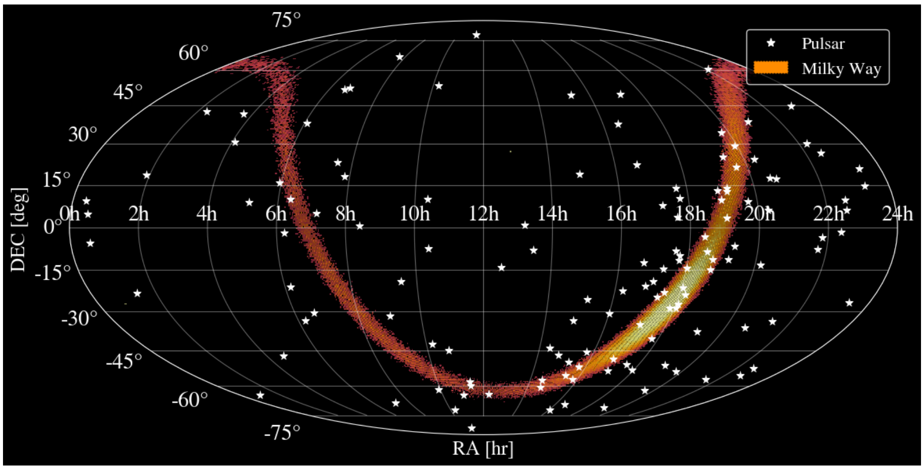
```
WARNING: Could not find pulsar distance for PSR J1903-705
1. Setting value to 1 with 20% uncertainty.
J1903+0327
J1903-7051
J1909-3744
J1910+1256
WARNING: Could not find pulsar distance for PSR J1911+134
7. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J1911-111
4. Setting value to 1 with 20% uncertainty.
J1911+1347
J1911-1114
WARNING: Could not find pulsar distance for PSR J1923+251
5. Setting value to 1 with 20% uncertainty.
J1918-0642
WARNING: Could not find pulsar distance for PSR J1933-621
1. Setting value to 1 with 20% uncertainty.
J1923+2515
J1933-6211
WARNING: Could not find pulsar distance for PSR J1944+090
7. Setting value to 1 with 20% uncertainty.
J1939+2134
WARNING: Could not find pulsar distance for PSR J1946+341
7. Setting value to 1 with 20% uncertainty.
J1944+0907
WARNING: Could not find pulsar distance for PSR J1946-540
3. Setting value to 1 with 20% uncertainty.
J1946+3417
J1946-5403
WARNING: Could not find pulsar distance for PSR J1955+290
8. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J2010-132
3. Setting value to 1 with 20% uncertainty.
J1955+2908
WARNING: Could not find pulsar distance for PSR J2017+060
3. Setting value to 1 with 20% uncertainty.
J2010-1323
WARNING: Could not find pulsar distance for PSR J2019+242
5. Setting value to 1 with 20% uncertainty.
J2017+0603
WARNING: Could not find pulsar distance for PSR J2033+173
4. Setting value to 1 with 20% uncertainty.
J2019+2425
WARNING: Could not find pulsar distance for PSR J2039-361
6. Setting value to 1 with 20% uncertainty.
J2033+1734
J2039-3616
WARNING: Could not find pulsar distance for PSR J2043+171
1. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J2055+382
9. Setting value to 1 with 20% uncertainty.
J2043+1711
J2055+3829
J2124-3358
J2129-5721
```

```
WARNING: Could not find pulsar distance for PSR J2150-032
6. Setting value to 1 with 20% uncertainty.
J2145-0750
WARNING: Could not find pulsar distance for PSR J2205+601
2. Setting value to 1 with 20% uncertainty.
J2150-0326
WARNING: Could not find pulsar distance for PSR J2214+300
0. Setting value to 1 with 20% uncertainty.
J2205+6012
J2214+3000
WARNING: Could not find pulsar distance for PSR J2222-013
7. Setting value to 1 with 20% uncertainty.
J2222-0137
WARNING: Could not find pulsar distance for PSR J2229+264
3. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J2234+061
1. Setting value to 1 with 20% uncertainty.
J2229+2643
J2234+0611
WARNING: Could not find pulsar distance for PSR J2234+094
4. Setting value to 1 with 20% uncertainty.
WARNING: Could not find pulsar distance for PSR J2236-552
7. Setting value to 1 with 20% uncertainty.
J2234+0944
WARNING: Could not find pulsar distance for PSR J2241-523
6. Setting value to 1 with 20% uncertainty.
J2236-5527
WARNING: Could not find pulsar distance for PSR J2302+444
2. Setting value to 1 with 20% uncertainty.
J2241-5236
J2302+4442
WARNING: Could not find pulsar distance for PSR J2322+205
7. Setting value to 1 with 20% uncertainty.
J2317+1439
WARNING: Could not find pulsar distance for PSR J2322-265
0. Setting value to 1 with 20% uncertainty.
J2322+2057
J2322-2650
```

2.3 Sky distribution of the pulsars

The millisecond pulsars (MSPs) are not uniformly distributed on the sky, they are predominantly found along the **Galactic plane**, reflecting both theit intrinsic distribution in the Milky Way and observational selection effects (sensitivity of radio telescopes, scattering from the ionized interstellar medium).

In [9]:
```
# Equatorial coordinates: "eq" ; Galactic coordinates: "g
coordtype = "eq"

plot_SkyMap(psrs, coordtype, show_mw=True, psrcolor = "w'
```

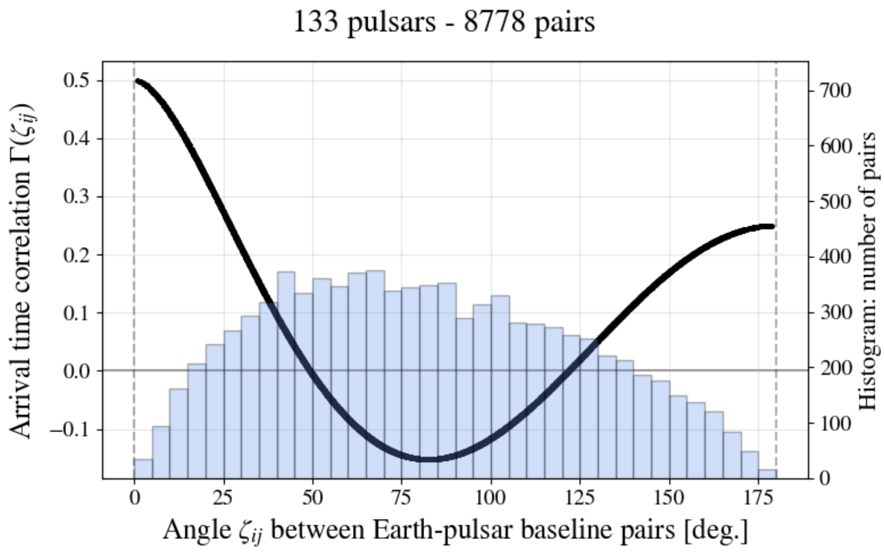


2.4 Pulsar pair distribution of the data set

To constrain the Arrival time correlations and search for the HD signature, we need to have a lot of pulsar pairs, so a lot of pulsars. Let us plot our coverage for the HD curve.

FYI, the number of pairs for n pulsars is $n(n-1)/2$

In [10…
```
# Play with pulsar numbers ; Max is 133 here (our full 
Npsrs = 133

# Reducing the number of pulsars to play with the curve
selpsrs = np.random.choice(psrs, size=Npsrs, replace=Fa

# Bin size for the histogram (in deg.)
bin_size_deg=5

# Plot !
plot_HD_from_psrlist(psrs=selpsrs, bin_size_deg=bin_siz
```

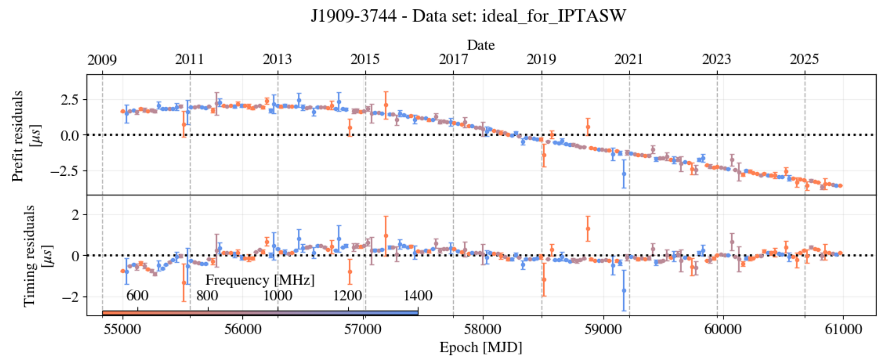


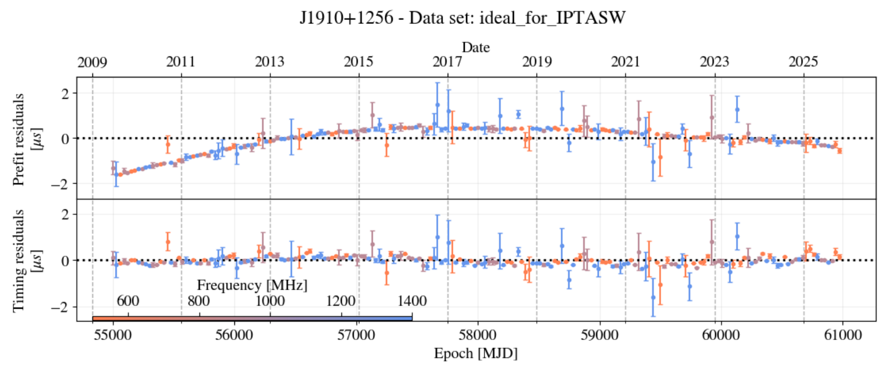
2.5 Prefit vs. Postfit residuals

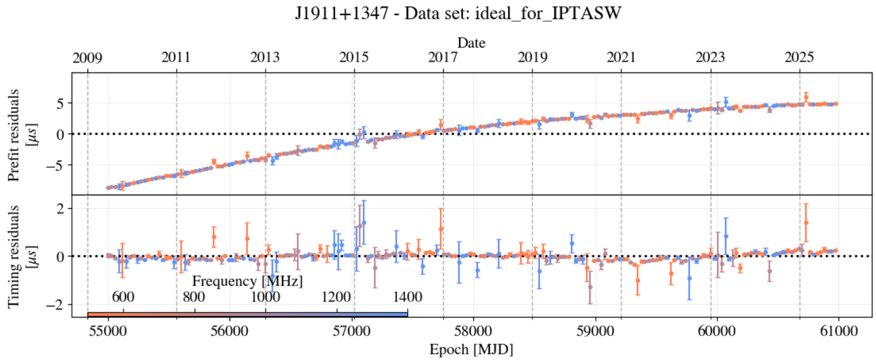
Let us now plot the timing residuals (before and after fitting for the timing model parameters)

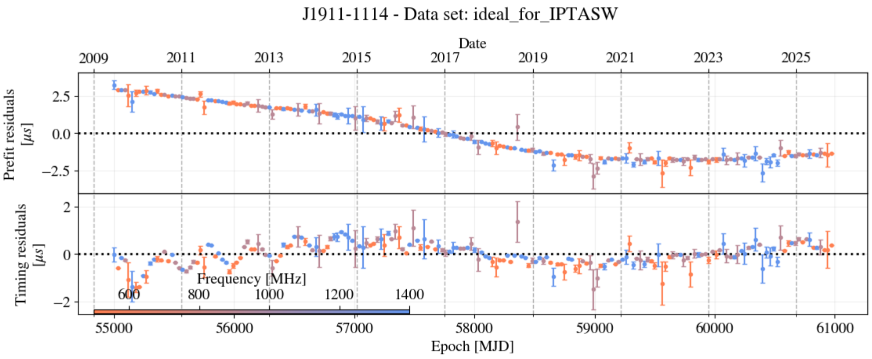
In [11…

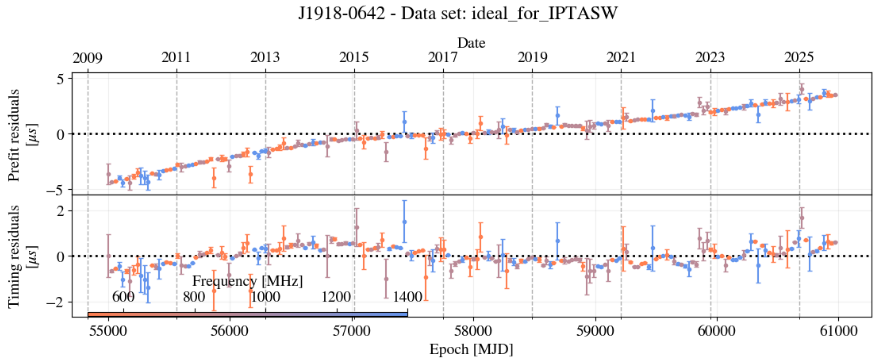
```
for psr in psrs[-35:-30]:
    plot_pulsar_timing(psr, prefit_res=prefit_res[psr.
#     plot_pulsar_timing(psr, plot_histo=True, addtitl
```











Some selected references related to Section 2

- Tempo2, a new pulsar timing package. II: The timing model and precision estimates (Edwards et al., 2006), The Tempo2 timing model
- libstempo: Python wrapper for Tempo2 (Vallisneri et al., 2020), reference for Libstempo

3. Modeling the noise and GWB

Useful functions

In [12…
```
def set_pta_enterprise(psrs, gwb_model, noise_model=

    # Choose a random subset of Npsrs pulsars
    if Npsrs is None:
        Npsrs = len(psrs)
    selpsrs = np.random.choice(psrs, size=Npsrs, re

    # Sort pulsar objects from names
    idxs = np.argsort([p.name for p in selpsrs])
    selpsrs = [psr for psr in selpsrs[idxs]]

    ###########################
    ## Set up the noise model
    ###########################

    ########################### Timing Model margina
    # Initiate the enterprise signal_collection obje
    tm = gp_signals.TimingModel()

    ########################### White noise
    ## EFAC - Here we don't consider EQUAD/ECORR, ju
    efac = white_noise_block(vary=False, tnequad=No

    ########################### Achromatic Red Noise
    rn = red_noise_block(psd="powerlaw", components=

    ########################### DM variations - Not
    dmgp = dm_noise_block(psd="powerlaw", component

    ########################### Common Red Signal
    # Common Red Signal uncorrelated power-law
    if gwb_model == "curn_pl":
        orf = None
        crn_psd = "powerlaw"
        crn_name = "gw_curn_pl"

    # Common Red Signal uncorrelated free-spectrum
    elif gwb_model == "curn_fs":
        orf = None
        crn_psd = "spectrum"
        crn_name = "gw_curn_fs"

    # Helling-Downs correlated power-law
    elif gwb_model == "hd_pl":
        orf = "hd"
        crn_psd = "powerlaw"
        crn_name = "gw_hd_pl"

    crn = common_red_noise_block(psd=crn_psd, compo

    ###########################
    ## Set up the enterprise SignalCollection object
    ###########################
    if noise_model=="ideal":
        signal = tm + efac + crn
    elif noise_model=="realistic":
        signal = tm + efac + rn + dmgp + crn

    model = [signal(psr) for psr in selpsrs]
```

```
    ###########################
    ## Set up the enterprise PTA object
    ###########################
    pta = signal_base.PTA(model)

    ########################### Fix EFAC values to 1
    params = {}
    for psrname in psrnames:
        parname = f"{psrname}_efac"
        parval = 1.
        params.update({parname:parval})

    pta.set_default_params(params)

    if verbose:
        print(f"PTA object set for {Npsrs} pulsars,

    return selpsrs, pta
```

3.1 The PTA Likelihood

In order to constrain models in PTA data, we build a likelihood function: the probability of observing the data given a chosen model and its parameter values.

The full PTA likelihood is a multivariate Gaussian over all pulsars jointly:

$$\mathcal{L}(\delta\mathbf{t} \mid \boldsymbol{\theta}) = \frac{1}{\sqrt{(2\pi)^n \det \mathbf{C}}} \exp\left(-\frac{1}{2}\delta\mathbf{t}^\top \mathbf{C}^{-1}\delta\mathbf{t}\right)$$

where $\delta\mathbf{t} = (\delta t_1, \ldots, \delta t_{N_p})$ is the concatenated residual vector over all pulsars, and $\mathbf{C}$ is the full covariance matrix.

In this tutorial, we consider three components: the timing model solution errors, the white noise (EFAC only) and the Gravitational Wave Background, all included in the covariance matrix as:

$$\mathbf{C} = \mathbf{C}^{\mathrm{TM}} + \mathbf{C}^{\mathrm{WN}} + \mathbf{C}^{\mathrm{GWB}}$$

Components of the covariance matrix

1. White noise $\mathbf{C}^{\mathrm{WN}}$

Each TOA has an independent measurement uncertainty σ_{TOA}, so the white noise covariance is purely diagonal:

$$\mathbf{C}_{ij}^{\mathrm{WN}} = \mathrm{EFAC}\, \sigma_{\mathrm{TOA}}^2(t_i)\; \delta_{ij} = \sigma_{\mathrm{TOA}}^2(t_i)\; \delta_{ij}$$

Here EFAC is fixed to 1 (its injected value), so it does not rescale the uncertainties.
No free parameters.

2. Timing model $\mathbf{C}^{\mathrm{TM}}$

Rather than fitting for timing model parameters explicitly, we marginalize over them analytically. See references as Haasteren & Vallisneri (2014) for more details.
No free parameters.

3. Gravitational Wave Background $\mathbf{C}^{\mathrm{GWB}}$

The GWB is modeled as a stationary Gaussian process with Fourier basis functions Φ at frequencies indexed with μ and ν. Its covariance between TOAs observed at times $t_{i,a}$ and $t_{j,b}$, respectively for pulsars a and b is:

$$\mathbf{C}_{ab}^{\mathrm{GWB}}(t_{i,a}, t_{jb}) = \Gamma_{ab} \cdot \sum_{\mu,\nu} \Phi_\mu(t_{i,a}) \Sigma_{\mu\nu} \Phi_\nu(t_{j,b})$$

where $\Sigma_{\mu\nu} = S(f_\mu) \cdot \delta_{\mu\nu}/T$, with $S(f)$ the GWB power spectral density (PSD) at Fourier frequency f, $\delta_{\mu\nu}$ the kronecker delta and T the total observing time span of the data set.

Γ_{ab} is the overlap reduction function (ORF), which encodes the spatial correlation between pulsar pairs. The ORF depends on the angular separation ζ_{ab} on the sky; for an isotropic GWB it takes the form of the Hellings-Downs curve (HD).

Setting $\Gamma_{ab} = \delta_{ab}$ instead gives a **common uncorrelated red noise (CURN)**, a common red process with no spatial correlations.

For the PSD model, the two choices considered in this tutorials are:

- **Power law** parametrized by an amplitude A and spectral index γ:

  $$S(f) = \frac{A^2}{12\pi^2}\left(\frac{f}{f_{\rm yr}}\right)^{-\gamma} f_{\rm yr}^{-3}$$

  For a GW-driven population of circular SMBHBs, $\gamma = 13/3$.

  The free parameters are $\{A, \gamma\}$.

- **Free spectrum**, parametrized by the PSD amplitude $S(f_k)$ at each Fourier frequency bin f_k, allowing a non-parametric reconstruction of the GWB spectrum.

The free parameters are $\{S(f_1), \ldots, S(f_{N_f})\}$.

In this tutorial, we consider a **CURN** ($\Gamma_{ab} = \delta_{ab}$), eith with a **power-law** or a **free spectrum** model for the Bayesian analysis. The HD model will be applied for the Optimal Statistics section.

3.2 How to compute it

Let us now compute the PTA likelihood (and priors for the Bayesian analysis) with Enterprise "by hand".

In [13…

```
# Initiate the enterprise
signal_collection object with the
TimingModel
tm =
gp_signals.TimingModel(use_svd=True)

########################### White
noise
## EFAC - Here we don't consider
EQUAD/ECORR, just for simplicity
efac_prior = parameter.Constant()
efac =
white_signals.MeasurementNoise(efac=

########################### Common
Red Signal
# Common Red Signal uncorrelated
power-law
orf = None # "hd"
crn_psd = "powerlaw" # "spectrum"
crn_name = "gw_curn_pl"

crn =
common_red_noise_block(psd=crn_psd,
components=30, prior='log-uniform',
orf=orf, name=crn_name)
```

```
###########################
## Set up the enterprise
SignalCollection object
###########################
signal = tm + efac + crn
model = [signal(psr) for psr in
psrs]

###########################
## Set up the enterprise PTA object
###########################
pta = signal_base.PTA(model)

########################### Fix
EFAC values to 1.
params = {}
for psrname in psrnames:
    parname = f"{psrname}_efac"
    parval = 1.
    params.update({parname:parval})

pta.set_default_params(params)


print(f"PTA object set for
{len(psrs)} pulsars, using
{str(orf)} as ORF and '{crn_psd}'
as PSD.")
```

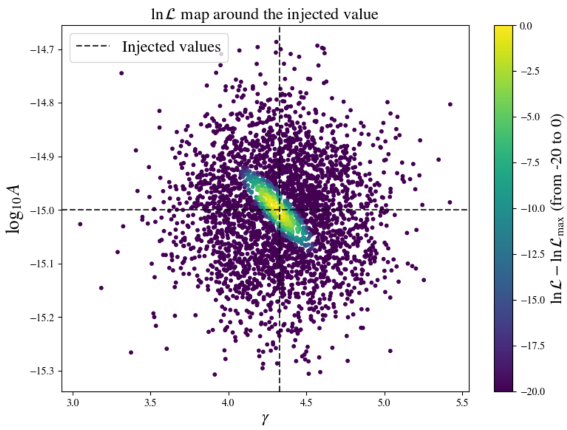
```
PTA object set for 133 pulsars, using
None as ORF and 'powerlaw' as PSD.
```

3.3 Playing with the PTA object

In [14…
```
y = np.array([4.33, -15])
cov = np.diag([.1, .01])

N = 3000

xs = np.zeros((3, N))
for i in range(N):
    print(f"{(i+1)} on {N}",
end="\r")
    g, A =
np.random.multivariate_normal(y,
cov=cov)

    x = {
        'gw_curn_pl_gamma':g,
        'gw_curn_pl_log10_A':A
    }

    xs[0,i] = g
    xs[1,i] = A
    xs[2,i] =
pta.get_lnlikelihood(x)
```

```
3000 on 3000
```

In [15…
```
fig, ax = plt.subplots(figsize=(8,
6))
sc = ax.scatter(xs[0], xs[1],
c=xs[2]-xs[2].max(), vmin=-20,
vmax=0, cmap='viridis', s=10)
cb = plt.colorbar(sc, ax=ax)
cb.set_label(label=r'$
\ln\mathcal{L} - \ln\mathcal{L}
_{\rm max}$ (from -20 to 0)',
size=16, labelpad=5)

# Mark injected values
ax.axvline(4.33, ls='--',
color='k', lw=1.5, label='Injected
values', alpha=.8)
ax.axhline(-15,  ls='--',
color='k', lw=1.5, alpha=.8)
ax.set_xlabel(r'$\gamma$',
fontsize=16)
ax.set_ylabel(r'$\log_{10} A$',
fontsize=18)
ax.set_title(r'$\ln\mathcal{L}$
map around the injected value',
fontsize=16)
ax.legend(fontsize=16)
plt.tight_layout()
plt.show()
```



In [16…

```
pta.summary()
```

```
Out[16]: 'enterprise v3.4.4,  Python v3.1
1.15\n===========================
==================================
===========================\n\nSi
gnal Name
Signal Class                   no.
Parameters      \n================
==================================
==================================
======\nJ0023+0923_linear_timing_m
odel_svd       TimingModel
0                    \n\nparams:\n_
__________________________________
__________________________________
_____________________\nJ0023+0923_
measurement_noise             Meas
urementNoise               0
\n\nparams:\nJ0023+0923_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0023+09
23_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0030+04
51_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0030+0451_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0030+0451_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0030+04
51_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0034-05
34_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
```

```
__________________________________
_\nJ0034-0534_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0034-0534_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0034-05
34_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0101-64
22_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0101-6422_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0101-6422_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0101-64
22_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0125-23
27_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0125-2327_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0125-2327_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0125-23
27_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
```

```
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0154+18
33_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0154+1833_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0154+1833_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ0154+18
33_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0218+42
32_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0218+4232_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0218+4232_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ0218+42
32_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0340+41
30_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0340+4130_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0340+4130_efac:Const
ant=1.0
```

```
\n________________________________
__________________________________
________________________\nJ0340+41
30_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0406+30
39_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n______________________
__________________________________
__________________________________
_\nJ0406+3039_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0406+3039_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0406+30
39_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0437-47
15_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n______________________
__________________________________
__________________________________
_\nJ0437-4715_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0437-4715_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0437-47
15_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0509+08
```

```
56_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0509+0856_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0509+0856_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0509+08
56_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0557+15
51_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0557+1551_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0557+1551_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0557+15
51_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0605+37
57_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0605+3757_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0605+3757_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0605+37
57_gw_curn_pl                    B
asisGP                         2
```

```
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0610-21
00_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0610-2100_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ0610-2100_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ0610-21
00_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0613-02
00_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0613-0200_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ0613-0200_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ0613-02
00_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0614-33
29_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
```

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_\nJ0614-3329_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ0614-3329_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0614-33
29_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0621+10
02_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0621+1002_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ0621+1002_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0621+10
02_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0636+51
28_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0636+5128_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ0636+5128_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0636+51
28_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
```

```
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0636-30
44_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0636-3044_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ0636-3044_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ0636-30
44_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0645+51
58_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0645+5158_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ0645+5158_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ0645+51
58_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ0709+04
58_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0709+0458_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ0709+0458_efac:Const
ant=1.0
\n__________________________________
```

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________________________________
_______________________\nJ0709+04
58_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n______________________________
________________________________
_______________________\nJ0711-68
30_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n____________________
________________________________
________________________________
_\nJ0711-6830_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0711-6830_efac:Const
ant=1.0
\n______________________________
________________________________
_______________________\nJ0711-68
30_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n______________________________
________________________________
_______________________\nJ0732+23
14_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n____________________
________________________________
________________________________
_\nJ0732+2314_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ0732+2314_efac:Const
ant=1.0
\n______________________________
________________________________
_______________________\nJ0732+23
14_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n______________________________
________________________________
_______________________\nJ0740+66
20_linear_timing_model_svd       T
```

```
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0740+6620_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0740+6620_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0740+66
20_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0751+18
07_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0751+1807_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0751+1807_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0751+18
07_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ0824+00
28_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ0824+0028_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0824+0028_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ0824+00
28_gw_curn_pl                    B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
```

```
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
________________________\nJ0900-31
44_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
____________________________________
____________________________________
_\nJ0900-3144_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0900-3144_efac:Const
ant=1.0
\n__________________________________
____________________________________
________________________\nJ0900-31
44_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
________________________\nJ0931-19
02_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
____________________________________
____________________________________
_\nJ0931-1902_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ0931-1902_efac:Const
ant=1.0
\n__________________________________
____________________________________
________________________\nJ0931-19
02_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
________________________\nJ0955-61
50_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
____________________________________
____________________________________
_\nJ0955-6150_measurement_noise
```

```
MeasurementNoise            0
\n\nparams:\nJ0955-6150_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ0955-61
50_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1012+53
07_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1012+5307_measurement_noise
MeasurementNoise            0
\n\nparams:\nJ1012+5307_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1012+53
07_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1012-42
35_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1012-4235_measurement_noise
MeasurementNoise            0
\n\nparams:\nJ1012-4235_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1012-42
35_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
```

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\n____________________________________
____________________________________
_________________________\nJ1017-71
56_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n______________________
____________________________________
____________________________________
_\nJ1017-7156_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1017-7156_efac:Const
ant=1.0
\n____________________________________
____________________________________
_________________________\nJ1017-71
56_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n____________________________________
____________________________________
_________________________\nJ1022+10
01_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n______________________
____________________________________
____________________________________
_\nJ1022+1001_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1022+1001_efac:Const
ant=1.0
\n____________________________________
____________________________________
_________________________\nJ1022+10
01_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n____________________________________
____________________________________
_________________________\nJ1024-07
19_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n______________________
____________________________________
____________________________________
_\nJ1024-0719_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1024-0719_efac:Const
ant=1.0
\n____________________________________
____________________________________
```

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________________________\nJ1024-07
19_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1036-83
17_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1036-8317_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1036-8317_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1036-83
17_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1045-45
09_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1045-4509_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1045-4509_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1045-45
09_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1101-64
24_linear_timing_model_svd        T
imingModel                      0
```

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\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1101-6424_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1101-6424_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1101-64
24_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1125+78
19_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1125+7819_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1125+7819_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1125+78
19_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1125-58
25_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1125-5825_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1125-5825_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1125-58
25_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
```

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oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1125-60
14_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1125-6014_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1125-6014_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1125-60
14_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1216-64
10_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1216-6410_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1216-6410_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1216-64
10_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1231-14
11_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1231-1411_measurement_noise
MeasurementNoise               0
```

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\n\nparams:\nJ1231-1411_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1231-14
11_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1312+00
51_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1312+0051_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1312+0051_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1312+00
51_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1327-07
55_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1327-0755_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1327-0755_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1327-07
55_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
```

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_________________________________
________________________\nJ1421-44
09_linear_timing_model_svd       T
imingModel                   0
\n\nparams:\n_____________________
_________________________________
_________________________________
_\nJ1421-4409_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1421-4409_efac:Const
ant=1.0
\n_______________________________
_________________________________
________________________\nJ1421-44
09_gw_curn_pl                    B
asisGP                       2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_______________________________
_________________________________
________________________\nJ1431-57
40_linear_timing_model_svd       T
imingModel                   0
\n\nparams:\n_____________________
_________________________________
_________________________________
_\nJ1431-5740_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1431-5740_efac:Const
ant=1.0
\n_______________________________
_________________________________
________________________\nJ1431-57
40_gw_curn_pl                    B
asisGP                       2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_______________________________
_________________________________
________________________\nJ1435-61
00_linear_timing_model_svd       T
imingModel                   0
\n\nparams:\n_____________________
_________________________________
_________________________________
_\nJ1435-6100_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1435-6100_efac:Const
ant=1.0
\n_______________________________
_________________________________
________________________\nJ1435-61
```

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00_gw_curn_pl                       B
asisGP                            2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1446-47
01_linear_timing_model_svd       T
imingModel                       0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1446-4701_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1446-4701_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1446-47
01_gw_curn_pl                       B
asisGP                            2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1453+19
02_linear_timing_model_svd       T
imingModel                       0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1453+1902_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1453+1902_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1453+19
02_gw_curn_pl                       B
asisGP                            2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1455-33
30_linear_timing_model_svd       T
imingModel                       0
\n\nparams:\n_____________________
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___________________________________
___________________________________
_\nJ1455-3330_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1455-3330_efac:Const
ant=1.0
\n_________________________________
___________________________________
________________________\nJ1455-33
30_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
___________________________________
________________________\nJ1513-25
50_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n______________________
___________________________________
___________________________________
_\nJ1513-2550_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1513-2550_efac:Const
ant=1.0
\n_________________________________
___________________________________
________________________\nJ1513-25
50_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
___________________________________
________________________\nJ1514-49
46_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n______________________
___________________________________
___________________________________
_\nJ1514-4946_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1514-4946_efac:Const
ant=1.0
\n_________________________________
___________________________________
________________________\nJ1514-49
46_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
```

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g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n___________________________________
___________________________________
________________________\nJ1518+49
04_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1518+4904_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1518+4904_efac:Const
ant=1.0
\n___________________________________
___________________________________
________________________\nJ1518+49
04_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n___________________________________
___________________________________
________________________\nJ1525-55
45_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1525-5545_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1525-5545_efac:Const
ant=1.0
\n___________________________________
___________________________________
________________________\nJ1525-55
45_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n___________________________________
___________________________________
________________________\nJ1543-51
49_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1543-5149_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1543-5149_efac:Const
```

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ant=1.0
\n__________________________________
__________________________________
________________________\nJ1543-51
49_gw_curn_pl                      B
asisGP                           2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1545-45
50_linear_timing_model_svd        T
imingModel                       0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1545-4550_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1545-4550_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1545-45
50_gw_curn_pl                      B
asisGP                           2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1547-57
09_linear_timing_model_svd        T
imingModel                       0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1547-5709_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1547-5709_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1547-57
09_gw_curn_pl                      B
asisGP                           2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
```

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________________________\nJ1600-30
53_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1600-3053_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1600-3053_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1600-30
53_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1603-72
02_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1603-7202_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1603-7202_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1603-72
02_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1614-22
30_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1614-2230_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1614-2230_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1614-22
30_gw_curn_pl                    B
```

```
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
________________________\nJ1629-69
02_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
____________________________________
____________________________________
_\nJ1629-6902_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1629-6902_efac:Const
ant=1.0
\n__________________________________
____________________________________
________________________\nJ1629-69
02_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
________________________\nJ1630+37
34_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
____________________________________
____________________________________
_\nJ1630+3734_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1630+3734_efac:Const
ant=1.0
\n__________________________________
____________________________________
________________________\nJ1630+37
34_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
________________________\nJ1640+22
24_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
____________________________________
```

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_________________________________
_\nJ1640+2224_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1640+2224_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1640+22
24_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1643-12
24_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
_________________________________
_\nJ1643-1224_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1643-1224_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1643-12
24_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1652-48
38_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
_________________________________
_\nJ1652-4838_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1652-4838_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1652-48
38_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
```

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\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1653-20
54_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1653-2054_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1653-2054_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1653-20
54_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1658-53
24_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1658-5324_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1658-5324_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1658-53
24_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1705-19
03_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1705-1903_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1705-1903_efac:Const
ant=1.0
```

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\n________________________________
__________________________________
________________________\nJ1705-19
03_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1708-35
06_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n______________________
__________________________________
__________________________________
_\nJ1708-3506_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1708-3506_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1708-35
06_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1710+49
23_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n______________________
__________________________________
__________________________________
_\nJ1710+4923_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1710+4923_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1710+49
23_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1713+07
```

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47_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1713+0747_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1713+0747_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1713+07
47_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1719-14
38_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1719-1438_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1719-1438_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1719-14
38_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1721-24
57_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1721-2457_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1721-2457_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1721-24
57_gw_curn_pl                    B
asisGP                         2
```

```
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1730-23
04_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1730-2304_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1730-2304_efac:Const
ant=1.0
\n__________________________________
___________________________________
________________________\nJ1730-23
04_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1732-50
49_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1732-5049_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1732-5049_efac:Const
ant=1.0
\n__________________________________
___________________________________
________________________\nJ1732-50
49_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1737-08
11_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
```

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_\nJ1737-0811_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1737-0811_efac:Const
ant=1.0
\n__________________________________
___________________________________
________________________\nJ1737-08
11_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1738+03
33_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1738+0333_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1738+0333_efac:Const
ant=1.0
\n__________________________________
___________________________________
________________________\nJ1738+03
33_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1741+13
51_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1741+1351_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1741+1351_efac:Const
ant=1.0
\n__________________________________
___________________________________
________________________\nJ1741+13
51_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
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pmax=7)
\n__________________________________
__________________________________
________________________\nJ1744-11
34_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1744-1134_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1744-1134_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1744-11
34_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1745+10
17_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1745+1017_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1745+1017_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1745+10
17_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1745-09
52_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1745-0952_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1745-0952_efac:Const
ant=1.0
\n__________________________________
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__________________________________
________________________\nJ1745-09
52_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1747-40
36_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1747-4036_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1747-4036_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1747-40
36_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1751-28
57_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1751-2857_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ1751-2857_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1751-28
57_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1757-53
22_linear_timing_model_svd        T
```

```
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1757-5322_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1757-5322_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1757-53
22_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1801-14
17_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1801-1417_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1801-1417_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1801-14
17_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1802-21
24_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1802-2124_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1802-2124_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1802-21
24_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
```

```
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1804-27
17_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1804-2717_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1804-2717_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1804-27
17_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1804-28
58_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1804-2858_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1804-2858_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1804-28
58_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1811-24
05_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1811-2405_measurement_noise
```

```
MeasurementNoise              0
\n\nparams:\nJ1811-2405_efac:Const
ant=1.0
\n_________________________________
__________________________________
________________________\nJ1811-24
05_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
__________________________________
________________________\nJ1825-03
19_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1825-0319_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1825-0319_efac:Const
ant=1.0
\n_________________________________
__________________________________
________________________\nJ1825-03
19_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
__________________________________
________________________\nJ1832-08
36_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1832-0836_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ1832-0836_efac:Const
ant=1.0
\n_________________________________
__________________________________
________________________\nJ1832-08
36_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
```

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\n___________________________________
___________________________________
_________________________\nJ1843-11
13_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1843-1113_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1843-1113_efac:Const
ant=1.0
\n___________________________________
___________________________________
_________________________\nJ1843-11
13_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n___________________________________
___________________________________
_________________________\nJ1843-14
48_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1843-1448_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1843-1448_efac:Const
ant=1.0
\n___________________________________
___________________________________
_________________________\nJ1843-14
48_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n___________________________________
___________________________________
_________________________\nJ1853+13
03_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1853+1303_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1853+1303_efac:Const
ant=1.0
\n___________________________________
___________________________________
```

```
________________________\nJ1853+13
03_gw_curn_pl                      B
asisGP                           2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1857+09
43_linear_timing_model_svd       T
imingModel                       0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1857+0943_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1857+0943_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1857+09
43_gw_curn_pl                      B
asisGP                           2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1902-51
05_linear_timing_model_svd       T
imingModel                       0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1902-5105_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ1902-5105_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1902-51
05_gw_curn_pl                      B
asisGP                           2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ1903+03
27_linear_timing_model_svd       T
imingModel                       0
```

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\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1903+0327_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1903+0327_efac:Const
ant=1.0
\n_________________________________
___________________________________
________________________\nJ1903+03
27_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
___________________________________
________________________\nJ1903-70
51_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1903-7051_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1903-7051_efac:Const
ant=1.0
\n_________________________________
___________________________________
________________________\nJ1903-70
51_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
___________________________________
________________________\nJ1909-37
44_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
___________________________________
___________________________________
_\nJ1909-3744_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1909-3744_efac:Const
ant=1.0
\n_________________________________
___________________________________
________________________\nJ1909-37
44_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
```

```
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1910+12
56_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n______________________
___________________________________
___________________________________
_\nJ1910+1256_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1910+1256_efac:Const
ant=1.0
\n__________________________________
___________________________________
________________________\nJ1910+12
56_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1911+13
47_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n______________________
___________________________________
___________________________________
_\nJ1911+1347_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1911+1347_efac:Const
ant=1.0
\n__________________________________
___________________________________
________________________\nJ1911+13
47_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\nJ1911-11
14_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n______________________
___________________________________
___________________________________
_\nJ1911-1114_measurement_noise
MeasurementNoise               0
```

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\n\nparams:\nJ1911-1114_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1911-11
14_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1918-06
42_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n______________________
__________________________________
__________________________________
_\nJ1918-0642_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1918-0642_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1918-06
42_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1923+25
15_linear_timing_model_svd        T
imingModel                     0
\n\nparams:\n______________________
__________________________________
__________________________________
_\nJ1923+2515_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1923+2515_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1923+25
15_gw_curn_pl                     B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
```

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____________________________________
_________________________\nJ1933-62
11_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n______________________
____________________________________
____________________________________
_\nJ1933-6211_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1933-6211_efac:Const
ant=1.0
\n__________________________________
____________________________________
_________________________\nJ1933-62
11_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
_________________________\nJ1939+21
34_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n______________________
____________________________________
____________________________________
_\nJ1939+2134_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1939+2134_efac:Const
ant=1.0
\n__________________________________
____________________________________
_________________________\nJ1939+21
34_gw_curn_pl                      B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
____________________________________
_________________________\nJ1944+09
07_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n______________________
____________________________________
____________________________________
_\nJ1944+0907_measurement_noise
MeasurementNoise                 0
\n\nparams:\nJ1944+0907_efac:Const
ant=1.0
\n__________________________________
____________________________________
_________________________\nJ1944+09
```

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07_gw_curn_pl                  B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1946+34
17_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1946+3417_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1946+3417_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1946+34
17_gw_curn_pl                  B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1946-54
03_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ1946-5403_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1946-5403_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ1946-54
03_gw_curn_pl                  B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ1955+29
08_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
```

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__________________________________
__________________________________
_\nJ1955+2908_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ1955+2908_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ1955+29
08_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2010-13
23_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2010-1323_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2010-1323_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2010-13
23_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2017+06
03_linear_timing_model_svd       T
imingModel                    0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2017+0603_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2017+0603_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2017+06
03_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
```

```
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
__________________________________
________________________\nJ2019+24
25_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2019+2425_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2019+2425_efac:Const
ant=1.0
\n_________________________________
__________________________________
________________________\nJ2019+24
25_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
__________________________________
________________________\nJ2033+17
34_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2033+1734_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2033+1734_efac:Const
ant=1.0
\n_________________________________
__________________________________
________________________\nJ2033+17
34_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
__________________________________
________________________\nJ2039-36
16_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2039-3616_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2039-3616_efac:Const
```

```
ant=1.0
\n__________________________________
__________________________________
________________________\nJ2039-36
16_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2043+17
11_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2043+1711_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ2043+1711_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ2043+17
11_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2055+38
29_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2055+3829_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ2055+3829_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ2055+38
29_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
```

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________________________\nJ2124-33
58_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2124-3358_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2124-3358_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2124-33
58_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2129-57
21_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2129-5721_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2129-5721_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2129-57
21_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2145-07
50_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2145-0750_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2145-0750_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2145-07
50_gw_curn_pl                    B
```

```
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2150-03
26_linear_timing_model_svd       T
imingModel                   0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2150-0326_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ2150-0326_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ2150-03
26_gw_curn_pl                     B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2205+60
12_linear_timing_model_svd       T
imingModel                   0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2205+6012_measurement_noise
MeasurementNoise              0
\n\nparams:\nJ2205+6012_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ2205+60
12_gw_curn_pl                     B
asisGP                        2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2214+30
00_linear_timing_model_svd       T
imingModel                   0
\n\nparams:\n_____________________
__________________________________
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________________________________
_\nJ2214+3000_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2214+3000_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2214+30
00_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2222-01
37_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2222-0137_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2222-0137_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2222-01
37_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2229+26
43_linear_timing_model_svd       T
imingModel                      0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2229+2643_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2229+2643_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2229+26
43_gw_curn_pl                    B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
```

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\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2234+06
11_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2234+0611_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2234+0611_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ2234+06
11_gw_curn_pl                      B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2234+09
44_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2234+0944_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2234+0944_efac:Const
ant=1.0
\n__________________________________
__________________________________
________________________\nJ2234+09
44_gw_curn_pl                      B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
__________________________________
________________________\nJ2236-55
27_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2236-5527_measurement_noise
MeasurementNoise                0
\n\nparams:\nJ2236-5527_efac:Const
ant=1.0
```

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\n_________________________________
_________________________________
________________________\nJ2236-55
27_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
_________________________________
________________________\nJ2241-52
36_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
_________________________________
_________________________________
_\nJ2241-5236_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ2241-5236_efac:Const
ant=1.0
\n_________________________________
_________________________________
________________________\nJ2241-52
36_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
_________________________________
________________________\nJ2302+44
42_linear_timing_model_svd        T
imingModel                      0
\n\nparams:\n_____________________
_________________________________
_________________________________
_\nJ2302+4442_measurement_noise
MeasurementNoise                  0
\n\nparams:\nJ2302+4442_efac:Const
ant=1.0
\n_________________________________
_________________________________
________________________\nJ2302+44
42_gw_curn_pl                     B
asisGP                          2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n_________________________________
_________________________________
________________________\nJ2317+14
```

```
39_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2317+1439_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2317+1439_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2317+14
39_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2322+20
57_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2322+2057_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2322+2057_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2322+20
57_gw_curn_pl                    B
asisGP                         2
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n________________________________
__________________________________
________________________\nJ2322-26
50_linear_timing_model_svd       T
imingModel                     0
\n\nparams:\n_____________________
__________________________________
__________________________________
_\nJ2322-2650_measurement_noise
MeasurementNoise               0
\n\nparams:\nJ2322-2650_efac:Const
ant=1.0
\n________________________________
__________________________________
________________________\nJ2322-26
50_gw_curn_pl                    B
asisGP                         2
```

```
\n\nBasis shape (Ntoas x N basis f
unctions): (200, 60)\nN selected t
oas: 200\n\nparams:\ngw_curn_pl_lo
g10_A:Uniform(pmin=-18, pmax=-11)
\ngw_curn_pl_gamma:Uniform(pmin=0,
pmax=7)
\n__________________________________
___________________________________
________________________\n========
===================================
===================================
==============\nTotal params: 135
\nVarying params: 2\nCommon params
: 266\nFixed params: 133\nNumber o
f pulsars: 133\n'
```

In [ ]:

Let us now perform a Bayesian analysis to constrain the GWB signal We will then use the function "set_pta_enterprise" to compute the PTA object

Some selected references related to Section 3

- The Nanohertz Gravitational Wave Astronomer (Taylor, 2021), very useful book about PTA, again related to this part.
- New advances in the Gaussian-process approach to pulsar-timing data analysis (van Haasteren & Vallisneri, 2014), reference to see

how the PTA likelihood is built and how the stochastic signals are modeled as Gaussian processes.

- ENTERPRISE: Enhanced Numerical Toolbox Enabling a Robust PulsaR Inference SuitE (Ellis, Siemens & Hazboun, 2019), reference for Enterprise software

4. Bayesian analysis

Useful functions

In [17…

```
# Module-level globals,
populated before forking,
inherited by workers
_GLOBAL_PTA = None
_GLOBAL_RUN_KWARGS = None

def
_single_PTMCMC_run(outdir):
    """Worker function: reads
pta and kwargs from globals,
no pickling needed."""
    pta = _GLOBAL_PTA
    nsamples    =
_GLOBAL_RUN_KWARGS["nsamples"]
    SCAMweight  =
_GLOBAL_RUN_KWARGS["SCAMweight
    AMweight    =
_GLOBAL_RUN_KWARGS["AMweight"]
    DEweight    =
_GLOBAL_RUN_KWARGS["DEweight"]
    overwrite_dir =
_GLOBAL_RUN_KWARGS["overwrite_

    if os.path.exists(outdir)
and not overwrite_dir:
        print(f"\nOutdir
already exists: {outdir}")
        return

    if os.path.exists(outdir)
and overwrite_dir:
```

```
        shutil.rmtree(outdir)
        os.makedirs(outdir)
        print(f"\nCleared and
recreated: {outdir}")
    elif not
os.path.exists(outdir):
        os.makedirs(outdir)
        print(f"\nCreated
folder: {outdir}")

    x0 =
np.hstack([p.sample() for p
in pta.params])
    ndim = len(x0)
    cov =
np.diag(np.ones(ndim) *
0.01**2)
    sampler = ptmcmc(ndim,
pta.get_lnlikelihood,
pta.get_lnprior, cov,
outDir=outdir)
    save_runtime_info(pta,
outdir=outdir)

    jp = JumpProposal(pta,
None, empirical_distr=None)

sampler.addProposalToCycle(jp.
5)
    sel_sig = {"red_noise":
10, "dm": 10, "gw": 30}
    for sig, val in
sel_sig.items():
        if any([sig in p for
p in pta.param_names]):

sampler.addProposalToCycle(jp.
val)

    print(f"\nPTMCMC run
started in '{outdir}' with
{nsamples} iterations.\n")
    sampler.sample(x0,
int(nsamples),
SCAMweight=SCAMweight,
AMweight=AMweight,
DEweight=DEweight)


def run_PTMCMC(outdir, pta,
nsamples=1e5, SCAMweight=30,
AMweight=15, DEweight=50,

overwrite_dir=False,
NRuns=1):

    global _GLOBAL_PTA,
_GLOBAL_RUN_KWARGS  # single
declaration at top of
function
```

```
    _GLOBAL_PTA = pta
    _GLOBAL_RUN_KWARGS =
dict(nsamples=nsamples,
SCAMweight=SCAMweight,

AMweight=AMweight,
DEweight=DEweight,

overwrite_dir=overwrite_dir)

    if NRuns == 1:

_single_PTMCMC_run(f"{outdir}'

    else:
        outdirs =
[f"{outdir}/ptmcmc_{i}" for i
in range(NRuns)]
        print(f"\nLaunching
{NRuns} parallel PTMCMC
runs...")
        ctx =
mp.get_context("fork")
        with
ctx.Pool(processes=NRuns) as
pool:

pool.map(_single_PTMCMC_run,
outdirs)
        print(f"\nAll {NRuns}
runs completed.")

def run_PTMCMC_old(outdir,
pta, nsamples=1e5,
SCAMweight=30, AMweight=15,
DEweight=50,
overwrite_dir=False):

    if os.path.exists(outdir)
and not overwrite_dir:
        print(f"\nOutdir
already exists: {outdir}")
    else:
        if
os.path.exists(outdir) and
overwrite_dir:

shutil.rmtree(outdir)

os.makedirs(outdir)
            print(f"\nCleared
and recreated: {outdir}")

        elif not
os.path.exists(outdir):

os.makedirs(outdir)
            print(f"\nCreated
folder: {outdir}")
```

```
########################
Generic Set-up part
        # Define the initial
point
        x0 =
np.hstack([p.sample() for p
in pta.params])

        # Initialize the
parameter covariance matrix
used by SCAM and AM Jump
Proposals
        ndim = len(x0)
        cov =
np.diag(np.ones(ndim) *
0.01**2)

        # Initialize the
PTMCMC objectz
        sampler =
ptmcmc(ndim,
pta.get_lnlikelihood,
pta.get_lnprior, cov,
outDir=outdir)

        # Save set-up
information

save_runtime_info(pta,
outdir=outdir)


########################
Additionals Jump Proposals
        # Set Jump Proposals
        jp =
JumpProposal(pta, None,
empirical_distr=None)

        # always add draw
from prior

sampler.addProposalToCycle(jp.
5)

        # Jump proposals from
priors of selected params
        sel_sig = {
            "red_noise":10,
            "dm":10,
            "gw":30
        }
        for sig, val in
sel_sig.items():
            if any([sig in p
for p in pta.param_names]):

sampler.addProposalToCycle(jp.
val)
```

```
########################
Sample !
        print(f"\nPTMCMC run
started with {nsamples}
iterations.")
        sampler.sample(x0,
int(nsamples),
SCAMweight=SCAMweight,
AMweight=AMweight,
DEweight=DEweight)

def read_chains(chaindir,
burnin=0.3):
    ch = np.loadtxt(chaindir
+ '/chain_1.txt')
    ch = ch[int(len(ch) *
burnin):]
    pars =
np.loadtxt(chaindir + '/
pars.txt', dtype='str')
    return ch, pars


def read_all_chains(outdir,
NRuns=None, burnin=0.3):
    chains, pars = [], None

    if NRuns is None:
        # Single run, no
subdirectory
        chains, pars =
read_chains(outdir,
burnin=burnin)
        return chains, pars

    if NRuns == 'auto':
        # Find all ptmcmc_{i}
folders automatically
        import os
        subdirs = sorted([
            d for d in
os.listdir(outdir)
            if
d.startswith('ptmcmc_') and
os.path.isdir(os.path.join(out
d))
        ])
        run_indices =
[int(d.split('_')[1]) for d
in subdirs]
    else:
        run_indices =
range(NRuns)

    for i in run_indices:
        chaindir =
f"{outdir}/ptmcmc_{i}"
        ch, p =
```

```
read_chains(chaindir,
burnin=burnin)
        chains.append(ch)
        if pars is None:
            pars = p

    chains =
np.concatenate(chains,
axis=0)
    return chains, pars

def compute_rho(log10_A,
gamma, f, T):
    """
    Converts from power to
residual RMS.
    """
    fyr = 1 / (365.25*86400)
    return
np.sqrt((10**log10_A)**2 /
(12.0*np.pi**2)
                   *
fyr**(gamma-3) * f**(-gamma)
/ T)
```

4.1 Parameter estimation on the CURN power-law PSD model

We infer the GWB parameters $\theta = \{A, \gamma\}$ from the data using Bayes' theorem:

$$p(\theta \mid \delta t) = \frac{p(\delta t \mid \theta)\, p(\theta)}{p(\delta t)}$$

where:

- $p(\delta t \mid \theta)$ is the **likelihood**: for our Gaussian noise model, it is a multivariate Gaussian in the timing residuals δt

- $p(\theta)$ is the **prior**: log-uniform on A (in practice it is uniform on $\log_{10} A$) and uniform on γ
- $p(\delta t)$ is the **Bayesian evidence**, used for model comparison via Bayes factors

We sample the posterior using **MCMC** via the `enterprise` + `PTMCMCSampler` packages.

> **Two GWB models are run here:**
>
> - **CURN power-law** (`curn_pl`): a *Common Uncorrelated Red Noise* with a power-law PSD, fast to sample, but ignores inter-pulsar spatial correlations.
> - **HD power-law** (`hd_pl`): the same power-law PSD but with the Hellings-Downs ORF enforced, this is the true

> GWB model, but computing the HD covariance matrix at every likelihood evaluation is much more expensive. We won't really use this one.

In [18…

```
overwrite_dir = False
gwb_model = "curn_pl"
outdir_curnpl = f"{datadir}/
chains/CURN_pl/"

nsamples = 1e5

selpsrs, pta =
set_pta_enterprise(psrs,
gwb_model, verbose=True)

if
os.path.exists(outdir_curnpl)
and not overwrite_dir:
    print(f"\nOutdir already
exists: {outdir_curnpl}")
else:
    if
os.path.exists(outdir_curnpl)
and overwrite_dir:

shutil.rmtree(outdir_curnpl)

os.makedirs(outdir_curnpl)
        print(f"\nCleared and
recreated: {outdir_curnpl}")

    elif not
os.path.exists(outdir_curnpl):

os.makedirs(outdir_curnpl)
        print(f"\nCreated
folder: {outdir_curnpl}")

    ########################
Generic Set-up part
    # Define the initial
point
    x0 =
np.hstack([p.sample() for p
in pta.params])
```

```
    # Initialize the
parameter covariance matrix
used by SCAM and AM Jump
Proposals
    ndim = len(x0)
    cov =
np.diag(np.ones(ndim) *
0.01**2)

    # Initialize the PTMCMC
objectz
    sampler = ptmcmc(ndim,
pta.get_lnlikelihood,
pta.get_lnprior, cov,
outDir=outdir_curnpl)

    # Save set-up information
    save_runtime_info(pta,
outdir=outdir_curnpl)

    #######################
Additionals Jump Proposals
    # Set Jump Proposals
    jp = JumpProposal(pta,
None, empirical_distr=None)

    # always add draw from
prior

sampler.addProposalToCycle(jp.
5)

    # Jump proposals from
priors of selected params
    sel_sig = {
        "gw":30
    }
    for sig, val in
sel_sig.items():
        if any([sig in p for
p in pta.param_names]):

sampler.addProposalToCycle(jp.
val)

    #######################
Sample !
    print(f"\nPTMCMC run
started with {nsamples}
iterations.")
    sampler.sample(x0,
int(nsamples), SCAMweight=30,
AMweight=15, DEweight=50)
```

```
PTA object set for 133 pulsars
, using 'ideal' noise model an
d 'curn_pl' CRS model.

Outdir already exists: /home/p
ulsar/Downloads/GWB-search/Dat
aSim/ideal_for_IPTASW//chains/
CURN_pl/
```

Let it run for a bit...

4.2 Parameter estimation on the HD power-law PSD model

You can also change the gwb_model to "hd_pl", and see how adding "HD" correlations makes it way longer !

In [19…

```
overwrite_dir = False
gwb_model = "hd_pl"
outdir_hdpl = f"{datadir}/
chains/HD_pl/"

nsamples = 1e5

selpsrs, pta =
set_pta_enterprise(psrs,
gwb_model, verbose=True)

if
os.path.exists(outdir_hdpl)
and not overwrite_dir:
    print(f"\nOutdir
already exists:
{outdir_hdpl}")
else:
    if
os.path.exists(outdir_hdpl)
and overwrite_dir:

shutil.rmtree(outdir_hdpl)

os.makedirs(outdir_hdpl)
        print(f"\nCleared
and recreated:
```

```
{outdir_hdpl}")

    elif not
os.path.exists(outdir_hdpl)

os.makedirs(outdir_hdpl)
        print(f"\nCreated
folder: {outdir_hdpl}")


#########################
Generic Set-up part
    # Define the initial
point
    x0 =
np.hstack([p.sample() for
p in pta.params])

    # Initialize the
parameter covariance
matrix used by SCAM and AM
Jump Proposals
    ndim = len(x0)
    cov =
np.diag(np.ones(ndim) *
0.01**2)

    # Initialize the
PTMCMC objectz
    sampler = ptmcmc(ndim,
pta.get_lnlikelihood,
pta.get_lnprior, cov,
outDir=outdir_hdpl)

    # Save set-up
information
    save_runtime_info(pta,
outdir=outdir_hdpl)


#########################
Additionals Jump Proposals
    # Set Jump Proposals
    jp = JumpProposal(pta,
None,
empirical_distr=None)

    # always add draw from
prior

sampler.addProposalToCycle(
5)

    # Jump proposals from
priors of selected params
    sel_sig = {
        "gw":30
    }
    for sig, val in
sel_sig.items():
```

```
        if any([sig in p
for p in
pta.param_names]):

sampler.addProposalToCycle(
val)


########################
Sample !
    print(f"\nPTMCMC run
started with {nsamples}
iterations.")
    sampler.sample(x0,
int(nsamples),
SCAMweight=30,
AMweight=15, DEweight=50)
```

```
PTA object set for 133 pulsa
rs, using 'ideal' noise mode
l and 'hd_pl' CRS model.

Outdir already exists: /hom
e/pulsar/Downloads/GWB-searc
h/DataSim/ideal_for_IPTASW//
chains/HD_pl/
```

Just stop it, it might finish next month...

The key takeaway is that recovering the full Hellings-Downs signal in a Bayesian framework requires dedicated high-performance computing (e.g. GPU-accelerated likelihoods or HPC clusters), not a single laptop core. In practice, PTA collaborations run these analyses over weeks on clusters and now use Discovery on GPUs.

4.3 Parameter estimation on the HD free-

spectrum PSD model.

Rather than constraining a single power-law, a **free-spectrum** model recovers the GWB power independently at each frequency bin. This gives a model-agnostic view of the spectrum.

In [20…

```
gwb_model = "curn_fs"
outdir_curnfs =
f"{datadir}/chains/
CURN_fs/"

selpsrs, pta =
set_pta_enterprise(psrs,
gwb_model, verbose=True)
run_PTMCMC(outdir_curnfs,
pta, nsamples=1e5,
SCAMweight=30,
AMweight=15, DEweight=50)
```

```
PTA object set for 133 pul
sars, using 'ideal' noise
model and 'curn_fs' CRS mo
del.

Outdir already exists: /ho
me/pulsar/Downloads/GWB-se
arch/DataSim/ideal_for_IPT
ASW//chains/CURN_fs/
```

4.4 Visualizing the recovered posterior distributions

Once the MCMC chains have converged, we read them back, discard the first 30% as **burn-in**, and visualize the posterior in two ways:

1. **Trace plots**: show the chain value of each parameter vs. iteration. A well-mixed chain should look like white noise around a stable mean, with no long-range drift.
2. **Corner plot**: shows the 1D and 2D marginal posteriors for all parameter pairs. Tight, unimodal contours centered near the injected values confirm a successful recovery.

Let us focus on the CURN power-law model in this part.

> **Injected values:** $\log_{10} A = -15$,

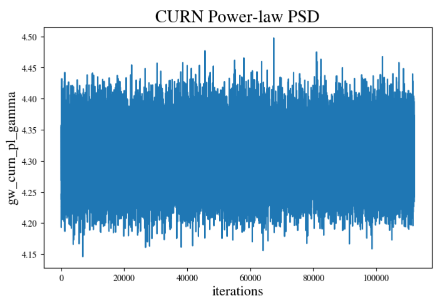
> $\gamma = 4.33$ (SMBHB power-law). The recovered posterior should be consistent with these.

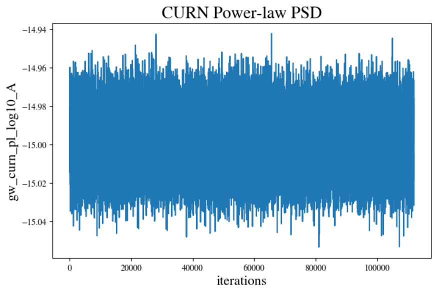
In [21…

```
outdir = '/home/pulsar/
Downloads/GWB-search/
DataSim/
ideal_for_IPTASW//chains/
CURN_pl_precomputed'
ch_curnpl, pars_curnpl =
read_all_chains(outdir,
NRuns='auto', burnin=0.3)
```

In [22…

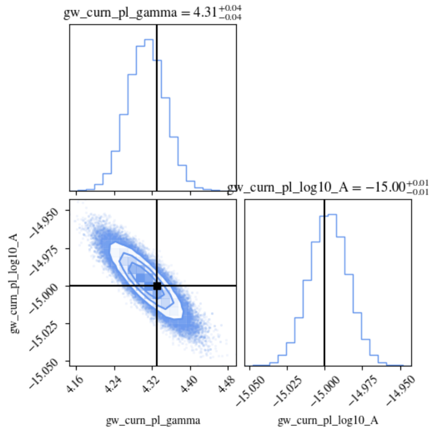
```
chain_to_plot = ch_curnpl
; pars_to_plot =
pars_curnpl ; titleplot =
"CURN Power-law PSD"

for i, p in
enumerate(pars_to_plot):

plt.figure(figsize=(8,5))
    plt.title(titleplot,
fontsize=20)

plt.plot(chain_to_plot[:,i

plt.xlabel("iterations",
fontsize=16)
    plt.ylabel(p,
fontsize=16)
    plt.show()
```





In [23…

```
corner.corner(chain_to_plo
labels=pars_to_plot,
truths=[4.33, -15],
color='cornflowerblue',
truth_color='k',
show_titles=True)
plt.show()
```



4.5 Visualizing the recovered GWB spectrum

We now overlay three representations of the recovered GWB spectrum on the same plot:

- **Injected power-law** (black): the ground truth used to simulate the data

- **Recovered CURN power-law** (coral): 1000 draws from the posterior of $(\log_{10} A, \gamma)$ converted to $\rho(f) = \sqrt{S_h(f)/T}$, the RMS timing residual contribution per frequency bin
- **Recovered CURN free-spectrum** (blue violins): the posterior of ρ at each frequency bin, each plotted as a violin. These are model-agnostic and should trace the injected power-law.

> **What to look for:** The free-spectrum violins should align with the injected power-law at low frequencies, and become consistent with the white noise

> level at high frequencies where the GWB signal is sub-dominant. The CURN power-law posterior should encompass the injected values.

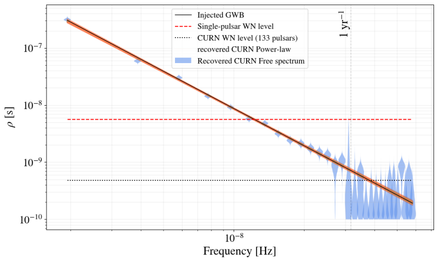
In [24…
```
outdir_curnpl =
f"{datadir}/chains/
CURN_pl_precomputed/"
outdir_curnfs =
f"{datadir}/chains/
CURN_fs_precomputed/"

ch_curnpl, pars_curnpl =
read_all_chains(outdir_cur
NRuns='auto', burnin=0.3)
ch_curnfs, pars_curnfs =
read_all_chains(outdir_cur
NRuns='auto', burnin=0.3)
```

In [25…
```
Tspan =
np.max([psr.toas.max()-
psr.toas.min() for psr in
psrs])
freqs = np.linspace(1/
Tspan, 30/Tspan, 30)
```

In [26…
```
plt.figure(figsize=(12,7))

#################
### Plot injected
powerlaw
rho_rn_inj =
compute_rho(log10_A=np.log
15), gamma=4.33, f=freqs,
T=Tspan)
plt.plot(freqs,
rho_rn_inj, color='k',
label="Injected GWB",
lw=1)

#################
### Get WN level, as
S(f)=2 * median(sigma)^2
* median(cadence)
(single-pulsar) and
```

```
S(f)=2 * median(sigma)^2
* median(cadence) / Npsr

Npsrs = len(psrs)
all_sigmas =
np.array([np.median(psr.to
for psr in psrs])
all_cadences =
np.array([np.median(psr.to
- psr.toas[::2]) for psr
in psrs])
sigma_eff =
np.median(all_sigmas)
cadence_eff =
np.median(all_cadences)
# Single-pulsar WN floor
rho_wn_single = np.sqrt(2
* sigma_eff**2 *
cadence_eff / Tspan)
plt.plot(freqs,
np.repeat(rho_wn_single,
len(freqs)), '--',
color='r', alpha=1,
label=f"Single-pulsar WN
level")
# CURN: averages over
Npsrs auto-correlations
rho_wn_curn =
rho_wn_single /
np.sqrt(Npsrs)
plt.plot(freqs,
np.repeat(rho_wn_curn,
len(freqs)), ':',
color='k', alpha=1,
label=f"CURN WN level
({Npsrs} pulsars)")

#################
### Plot powerlaw

# Define the indices to
use from the chain
N = 1000
idxs =
np.random.choice(range(len
size=N, replace=False)
As = ch_curnpl[:,
list(pars_curnpl).index('g
Gs = ch_curnpl[:,
list(pars_curnpl).index('g

for i in range(N):
    rho_rn =
compute_rho(log10_A=As[i],
gamma=Gs[i], f=freqs,
T=Tspan)
    if i==0:
        plt.plot(freqs,
rho_rn, color='coral',
alpha=.02,
```

```
label="recovered CURN
Power-law", zorder=1)
    else:
        plt.plot(freqs,
rho_rn, color='coral',
alpha=.02, zorder=1)

#################
### Plot free-spectrum

rhos = ch_curnfs[:,:-4]
parts =
plt.violinplot(10**rhos,
positions=freqs,
widths=freqs*0.07,
showextrema=False)
for pc in
parts['bodies']:

pc.set_facecolor('cornflow
    #
pc.set_edgecolor('black')
    pc.set_alpha(0.6)
    pc.set_zorder(0)

handles, labels =
plt.gca().get_legend_handl
handles.append(Patch(facec
alpha=0.6,
label="Recovered CURN
Free spectrum"))
labels.append("Recovered
CURN Free spectrum")

#################
### Others
plt.legend(handles=handles
labels=labels,
fontsize=14)
plt.axvline(1/86400/365.25
c='k', ls='--', lw=1,
alpha=.2)
plt.xscale('log')
plt.yscale('log')
plt.xlabel("Frequency
[Hz]", fontsize=20)
plt.ylabel(r"$\rho$ [s]",
fontsize=20)
plt.tick_params(labelsize=
plt.text(1/86400/365.25 -
3.9e-9, 1.7e-7, "1 $
\mathrm{yr}^{-1}$",
rotation=90, fontsize=20)
plt.grid(which='both',
alpha=.2)
plt.show()
```



Some selected references related to Section 4

- The Nanohertz Gravitational Wave Astronomer (Taylor, 2021), very useful book about PTA... again related to this part !!
- The International Pulsar Timing Array second data release: Search for an isotropic gravitational wave background (Antoniadis et al. 2022), GWB search with the IPTA DR2
- Comparing Recent Pulsar Timing Array Results on the Nanohertz Stochastic Gravitational-wave Background (Agazie et al. 2025), Comparison of results from

multiple PTAs that were published in 2023

- The second data release from the European Pulsar Timing Array — III. Search for gravitational wave signals (Antoniadis et al., EPTA (2023), GWB search with the EPTA DR2 and InPTA
- The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background (Agazie et al., 2023), GWB search with the NANOGrav 15yr
- Search for an Isotropic Gravitational-wave Background with the Parkes Pulsar Timing Array (Reardon et al., 2023), GWB search with PPTA DR3
- Searching for the Nano-Hertz Stochastic Gravitational Wave Background with the Chinese Pulsar Timing Array (Xu et al.,, 2023), GWB

search with the CPTA DR1
- The MeerKAT Pulsar Timing Array: the first search for gravitational waves with the MeerKAT radio telescope (Miles et al., 2025), GW search with MeerKAT
- PTMCMCSampler: Parallel tempering MCMC sampler (Ellis & van Haasteren, 2017), reference for PTMCMCSampler

5. Optimal Statistics (OS)

Useful functions

In [27…

```
# This function computes the weighted average and uncertainty
# for a set of values (rho) with associated uncertainties (sig).
# It returns the weighted average and the corresponding standard error.
def weightedavg(rho, sig):
    weights, avg = 0., 0.
```

```
    for r,s in
zip(rho,sig):
        weights +=
1./(s*s)
        avg += r/
(s*s)

    return avg/
weights, np.sqrt(1./
weights)


# This function bins
the cross-correlation
data (rho) and
associated
uncertainties (sig)
# according to
angular separation
values (xi), using
bins defined by zeta.
# Each bin spans 10
degrees in angle. It
returns the binned
weighted average
# cross-correlation
and corresponding
uncertainties.
def
bin_crosscorr_npairs_t
xi, rho, sig):
    """
    Bin cross-
correlations into
equal-number of
pulsar pairs
    """

    rho_avg, sig_avg
=
np.zeros(len(zeta)),
np.zeros(len(zeta))

    for i,z in
enumerate(zeta[:-1]):
        myrhos,
mysigs = [], []
        for x,r,s in
zip(xi,rho,sig):
            if x >= z
and x < (z+10.):

myrhos.append(r)

mysigs.append(s)
        rho_avg[i],
sig_avg[i] =
weightedavg(myrhos,
mysigs)
```

```
    return rho_avg,
sig_avg

def
bin_crosscorr_npairs(x
rho, sig,
NPairPerBin=10):
    """
    Bin cross-
correlations into
equal number of
pulsar pairs
    """
    # sort
    idx       =
np.argsort(xi)
    xi_sorted   =
xi_rad[idx]
    rho_sorted  =
rho[idx]
    sig_sorted  =
sig[idx]

    xi_mean, xi_err,
rho_avg, sig_avg =
[], [], [], []
    i = 0
    while i <
len(xi_sorted):

xi_mean.append(np.mean

xi_err.append(np.std(x
        r, s =
weightedavg(rho_sorted
sig_sorted[i:i+NPairPe

rho_avg.append(r)

sig_avg.append(s)
        i +=
NPairPerBin

    return
(np.array(xi_mean),
np.array(xi_err),
np.array(rho_avg),
np.array(sig_avg))

def
bin_crosscorr_binwidth
rho, sig,
bin_size_deg=10.0):
    """
    Bin cross-
correlations into
equal width angular
bins.

    Parameters
```

```
    ----------
    xi_rad      :
array, angular
separations in
radians
    rho         :
array, cross-
correlations
(normalized)
    sig         :
array, uncertainties
    bin_size_deg:
float, bin width in
degrees

    Returns
    -------
    xi_mean, xi_err,
rho_avg, sig_avg :
arrays of shape
(nbins,)
    """
    # sort
    idx        =
np.argsort(xi)
    xi_sorted   =
xi_rad[idx]
    rho_sorted  =
rho[idx]
    sig_sorted  =
sig[idx]

    bin_size_rad =
bin_size_deg * np.pi
/ 180

    xi_deg =
xi_sorted

    bin_edges  =
np.arange(0, np.pi +
bin_size_rad,
bin_size_rad)
    bin_centers = 0.5
* (bin_edges[:-1] +
bin_edges[1:])

    xi_mean, xi_err,
rho_avg, sig_avg =
[], [], [], []
    for lo, hi in
zip(bin_edges[:-1],
bin_edges[1:]):
        mask =
(xi_deg >= lo) &
(xi_deg < hi)

        if mask.sum()
== 0:
            continue
```

```
# skip empty bins


xi_mean.append(np.mean

xi_err.append(np.std(x
        r, s =
weightedavg(rho_sorted
sig_sorted[mask])

rho_avg.append(r)

sig_avg.append(s)

    return
(np.array(xi_mean),
np.array(xi_err),
np.array(rho_avg),
np.array(sig_avg))

def
get_histo_equal_Npairs
NPairPerBin):
    idx       =
np.argsort(xi_mean)
    xi_sorted   =
xi_mean[idx]
    bin_counts  =
[NPairPerBin] *
len(xi_sorted)

    # Estimate bin
edges as midpoints
between consecutive
xi_mean values
    bin_edges
=
np.zeros(len(xi_sorted
+ 1)
    bin_edges[1:-1] =
(xi_sorted[:-1] +
xi_sorted[1:]) / 2
# midpoints between
consecutive means
    bin_edges[0]
= xi_sorted[0] - 0.5
* (xi_sorted[1]  -
xi_sorted[0])   #
extrapolate left
    bin_edges[-1]
= xi_sorted[-1] + 0.5
* (xi_sorted[-1] -
xi_sorted[-2])  #
extrapolate right
    bin_widths
= np.diff(bin_edges)
    bin_centers
= bin_edges[:-1] +
bin_widths / 2  #
centers from edges,
```

```
not xi_mean

    return
bin_centers,
bin_widths,
bin_counts

def
get_histo_equal_BinWid
bin_size_deg):
    xi_deg = xi_rad *
180 / np.pi

    bin_edges   =
np.arange(0, 180 +
bin_size_deg,
bin_size_deg)  #
edges from 0 to 180
    bin_centers = 0.5
* (bin_edges[:-1] +
bin_edges[1:])
# centers at 5, 15,
25, ...
    bin_counts, _ =
np.histogram(xi_deg,
bins=bin_edges)
# use actual edges

    return
bin_centers * np.pi /
180, bin_counts

def GPfit(x, y, yerr,
x_output):
    kernel =
RBF(length_scale=30)
+ WhiteKernel()
    gp     =
GaussianProcessRegress
alpha=yerr**2)

gp.fit(x.reshape(-1,1)
y)

    y_fit, y_std_gp =
gp.predict(x_smooth.re
1), return_std=True)

    # Interpolate
measurement noise
onto smooth grid
    yerr_smooth =
np.interp(x_smooth,
x, yerr)

    # Total
uncertainty = GP
epistemic +
measurement noise in
quadrature
```

```
    y_std =
np.sqrt(y_std_gp**2 +
yerr_smooth**2)

    return y_fit,
y_std

def
build_Q_matrix_defiant
params):
    """
    Build the
normalized Q matrix
such that OS = d^T Q
d for d ~ N(0,I).
    Follows the
Hazboun et al.
construction
(QtildeIJ block
matrix).
    """
    npsr =
OS_obj.npsr
    nfreq =
OS_obj.nfreq
    ngw = 2 * nfreq

    # GW spectrum
    phi =
OS_obj._get_phi(params
# shape (ngw,)
    sPhi =
np.sqrt(phi)

    # ORF values for
HD
    orf_full =
OS_obj.orf_matrix[0]
# shape (npsr, npsr)
    pairs = [(i, j)
for i in range(npsr)
for j in range(i+1,
npsr)]
    orfs =
np.array([orf_full[i,
j] for (i, j) in
pairs])

    # noise-
marginalized GW
overlap matrices
    # FCF[i] = F^T
C_i^{-1} F
(marginalized over
timing model)
    FCFs = []
    for i in
range(npsr):
        FNF    =
OS_obj._get_FNF(i,
```

```
params)   # (ngw,
ngw)
        FNT    =
OS_obj._get_FNT(i,
params)   # (ngw,
ntim)
        TNT    =
OS_obj._get_TNT(i,
params)   # (ntim,
ntim)
        phiinv =
OS_obj._get_phiinv(i,
params) # (ntim,
ntim)
        inner  =
phiinv + TNT
        FCF    = FNF
- FNT @
np.linalg.solve(inner,
FNT.T)

FCFs.append(FCF)

    # P_i = sPhi @
FCF[i] @ sPhi  (= D_i
in Hazboun notation)
    Ps = [sPhi[:,
None] * FCF *
sPhi[None, :] for FCF
in FCFs]

    # Cholesky
factors: P_i = L_i
L_i^T
    Ls = []
    for P in Ps:
        P = 0.5 * (P
+ P.T)
        eps = 1e-10 *
np.trace(P) /
P.shape[0]
        L =
np.linalg.cholesky(P
+ eps * np.eye(ngw))
        Ls.append(L)

    # normalization:
bottom = sum_{i<j}
tr(Pinv_i S_ij Pinv_j
S_ij^T)
    # in our
notation: bottom =
sum_{i<j} orf^2 *
tr(P_i P_j)
    # = sum_{i<j}
orf^2 * tr(L_i^T L_j
L_j^T L_i)
    bottom = 0.0
    for w, (i, j) in
zip(orfs, pairs):
```

```
        LiTLj =
Ls[i].T @ Ls[j]
# (ngw, ngw)
        bottom +=
w**2 *
np.trace(LiTLj.T @
LiTLj)

#     norm = 1.0 /
np.sqrt(bottom)
    norm = 1.0 / (2.0
* np.sqrt(bottom))
#     print(f'norm:
{norm:.6f}, bottom:
{bottom:.6f}')

    # build block Q
matrix
    cnt = npsr * ngw
    inds = [slice(i *
ngw, (i + 1) * ngw)
for i in range(npsr)]
    Q =
np.zeros((cnt, cnt))

    for w, (i, j) in
zip(orfs, pairs):
        # Q_ij = norm
* orf_ij * L_i^T L_j
(off-diagonal block)
        Bij = norm *
w * (Ls[i].T @ Ls[j])
        Q[inds[i],
inds[j]] += Bij
        Q[inds[j],
inds[i]] += Bij.T

    # sanity checks
#     print(f'Q
symmetric:
{np.max(np.abs(Q -
Q.T)):.2e}')
#     print(f'tr(Q):
{np.trace(Q):.6f}
# should be 0')
#
print(f'sqrt(2*tr(Q^2)
{np.sqrt(2*np.trace(Q@
# should be 1')

    return Q

def
gx2_survival_hybrid(sn
eigs,
crossover_snr=0.0):
    """
    Use saddlepoint
(Lugannani-Rice) for
all SNR values.
```

```
    Imhof is
unreliable with the
current eigenvalue
set.
    The saddlepoint
is inaccurate near
SNR~0 but exact in
the tail,
    which is the only
region that matters
for p-value
reporting.
    """
    from scipy import
optimize

    eigs =
np.array(eigs)
    t_max = 0.999 /
(2 * np.max(eigs))
    t_min = 0.999 /
(2 * np.min(eigs)) if
np.min(eigs) < 0 else
-10.0

    def K(t):
        return -0.5 *
np.sum(np.log(1 - 2 *
eigs * t))
    def K1(t):
        return
np.sum(eigs / (1 - 2
* eigs * t))
    def K2(t):
        return
np.sum(2 * eigs**2 /
(1 - 2 * eigs *
t)**2)

    results = []
    methods = []

    for s in
snr_values:
        try:
            t_hat =
optimize.brentq(lambda
t: K1(t) - s, t_min,
t_max,

xtol=1e-12,
maxiter=1000)
            w =
np.sign(t_hat) *
np.sqrt(2 * (t_hat *
s - K(t_hat)))
            u_sp =
t_hat *
np.sqrt(K2(t_hat))
            if
```

```
np.abs(w) > 1e-6:
                p =
float(stats.norm.sf(w)
+ stats.norm.pdf(w) *
(1/w - 1/u_sp))
                p =
np.clip(p, 0, 1)
            else:
                p =
0.5
        except
Exception as e:
            p =
np.nan

results.append(p)

methods.append('saddle

    return
np.array(results),
methods
```

In [28…

```
inj_params = {

f'gw_curn_pl_gamma':4.

f'gw_curn_pl_log10_A':
-15
}
```

5.1 Motivation and description

The Bayesian approach is powerful but computationally expensive, especially when spatial correlations (HD ORF) are included. The **Optimal Statistic (OS)** is a frequentist cross-correlation estimator that:

- Is **fast** to compute (no MCMC required for the HD step itself)
- Provides an **amplitude estimate** $\hat{A}^2$ and its uncertainty $\sigma(\hat{A}^2)$
- Enables a **frequentist significance** estimate via the signal-to-noise ratio $\hat{\rho} = \hat{A}^2/\sigma(\hat{A}^2)$
- Allows per-pair **cross-correlation recovery**, directly tracing the Hellings-Downs curve

The OS estimator for the GWB amplitude squared $\hat{A}^2$ is:

$$\hat{A}^2 = \frac{\sum_{a<b} \delta t_a^T C_a^{-1} \tilde{S}_{ab} \ldots}{\sum_{a<b} \mathrm{tr}\Big(C_a^{-1} \tilde{S}_{ab} \ldots}$$

where:

- δt_a are the timing residuals of pulsar a
- C_a is the noise covariance matrix of pulsar a (including white noise and pulsar-intrinsic red noise)
- $\tilde{S}_{ab}(f) = \Gamma_{ab} \cdot S_h$ is the expected cross-power, with Γ_{ab} the HD coefficient for pair (a, b)

In the noise-dominated (weak signal) regime, $\langle \hat{A}^2 \rangle = 0$, while the **variance** of the estimator is:

$$\sigma(\hat{A}^2) = \Bigg[\sum_{a<b} \mathrm{tr}\Big(C_a^{-1} \ldots$$

Key subtlety: the noise marginalization The single-parameter OS above uses fixed noise parameters. In practice, the noise covariance C_a depends on estimated white-noise and red-noise parameters, which themselves carry uncertainty. The **Noise-Marginalized OS (NMOS)** marginalizes over this uncertainty by drawing noise parameters from the posterior of a noise-only

> MCMC chain and averaging the OS over those draws, this is what `compute_OS(N=1(` `...)` does below.

5.2 Amplitude recovery and signal-to-noise ratio

We now build the OS object using `defiant.OptimalSt` feeding it the CURN power-law MCMC chain so that the noise marginalization can draw from the posterior.

The cells below compute the **Noise-Marginalized OS (NMOS)** with $N = 100$ noise draws. Each draw gives a different $\hat{A}^2$ estimate; the resulting distribution captures both the statistical uncertainty of the OS and the propagated noise-parameter uncertainty.

Output of `compute_OS(N=1(`

- `A2` : array of $\hat{A}^2$ values, one per noise draw
- `A2s` : corresponding $\sigma(\hat{A}^2)$ values
- `idx` : indices of the noise draws used

In [29…

```
# Since we created
our PTA object with
'gw' as the name,
make sure to set
that!
gwb_model = "curn_pl"
outdir_curnpl =
f"{datadir}/chains/
CURN_pl_precomputed/"
```

```
selpsrs, pta =
set_pta_enterprise(psr
gwb_model="curn_pl",
verbose=True)
#
run_PTMCMC(outdir_curn
pta, nsamples=1e5,
SCAMweight=30,
AMweight=15,
DEweight=50,
overwrite_dir=True,
NRuns=10)
```

```
PTA object set for 133
pulsars, using 'ideal'
noise model and 'curn_
pl' CRS model.
```

In [30…

```
ch, pars =
read_all_chains(outdir
NRuns='auto')
os_obj =
OptimalStatistic(psrs,
pta=pta, chain=ch,
param_names=list(pars)
gwb_name='gw_curn_pl',
orfs=['hd'])
```

In [31…

```
#
os_obj.set_orf(['hd'])

# When N>1, params is
ignored. The default
value of params is
None, so we are good!
output_NMOS =
os_obj.compute_OS(N=10
return_pair_vals=False
A2, A2s, idx =
(output_NMOS[k] for k
in
['A2','A2s','idx'])
```

```
NMOS Iters: 100%|
██████████████████████
████████████████████|
100/100 [00:15<00:00,
6.50it/s]
```

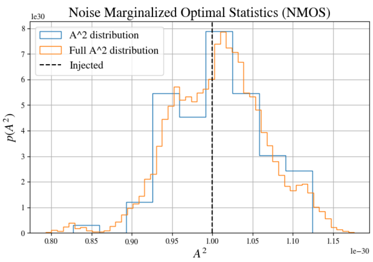
In [32…

```
# A function to
implement uncertainty
sampling to account
for underlying
uncertainty in the
optimal statistic A^2
full_A2 =
utils.uncertainty_samp
```

In [33…

```
plt.figure(figsize=(8,
```

```
plt.hist(A2,bins='auto
density=True,
label='A^2
distribution')
plt.hist(full_A2,bins=
density=True,
label='Full A^2
distribution')

plt.axvline(10**(2*(-1

plt.title('Noise
Marginalized Optimal
Statistics (NMOS)',
fontsize=18)
plt.xlabel('$A^2$',
fontsize=16)
plt.ylabel('$p(A^2)$',
fontsize=16)
plt.grid()
plt.legend(fontsize=14
plt.show()
```



5.3 Per-pair cross-correlations

Rather than a single amplitude, we can compute the OS **per angular separation bin**, recovering the shape of the inter-pulsar correlation as a function of ζ.

For each angular

bin $[\zeta_i, \zeta_{i+1}]$, we define a binned ORF $\hat{\Gamma}_i(\zeta_{ab})$ and compute the amplitude in that bin:

$$\hat{A}_i^2 = \frac{\sum_{a<b,\, \zeta_{ab} \in \text{bin } i} \delta}{\sum_{a<b,\, \zeta_{ab} \in \text{bin } i} \text{tr}}$$

Plotting $\hat{A}_i^2$ vs. ζ_i and comparing to the theoretical HD curve $\Gamma(\zeta)$ is the **most direct visual evidence** for the spatial correlations expected from a GWB.

Let us start with the enterprise_ext method

In [34…

```
## Here we compute the Optimal Statistic and the per-pair cross-correlations
OS_ee = ostat.OptimalStatist
pta=pta, orf='hd')

# xi: angular separation [rad] for each pulsar pair
# rho: correlation coefficient for each pulsar pair
# sig: 1-sigma uncertainty on correlation
```

```
coefficient for
each pulsar pair.
# OS: Optimal
statistic value
(units of A_gw^2)
# OS_sig: 1-sigma
uncertainty on OS
xi, rho, sig, OS,
OS_sig =
OS_ee.compute_os(pa

# Normalize by OS
to get correlation
in [-1, 1]
rho_norm = rho /
OS
sig_norm  = sig /
OS
```

In [35…

```
# adjust as wanted

plotype = 1 # 1 =
Equal Npairs, 2 =
Equal Bin Width

NPairPerBin = 200
# adjust as wanted
- Used if
plotype==1
bin_widths = 5 #
degree - adjust as
wanted - Used if
plotype==2

##############

if plotype == 1:
    xi_mean,
xi_err, rho_avg,
sig_avg =
bin_crosscorr_npair
rho_norm,
sig_norm,
NPairPerBin=NPairPe
    bin_centers,
bin_widths,
bin_counts =
get_histo_equal_Npa
NPairPerBin)
elif plotype == 2:
    xi_mean,
xi_err, rho_avg,
sig_avg =
bin_crosscorr_binwi
rho_norm,
sig_norm,
bin_size_deg=bin_wi
    bin_centers,
bin_counts =
get_histo_equal_Bin
```

```
bin_widths)
    bin_widths *=
np.pi / 180
```

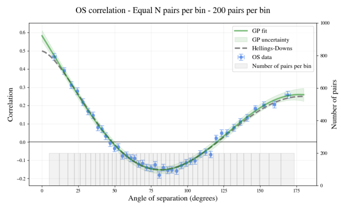
In [36…

```
fig, ax1 =
plt.subplots(figsize
6))

if plotype==1:
    add_title =
f"Equal N pairs
per bin -
{NPairPerBin}
pairs per bin"
if plotype==2:
    add_title =
f"Equal bin widths
- {bin_widths *
180/np.pi:.1f}
degrees per bin"

fig.suptitle(f"OS
correlation -
{add_title}",
fontsize=18)

# ---- Left y-
axis: correlations
----
###################
ax1.errorbar(xi_mear
np.pi, rho_avg,
xerr=xi_err*180/
np.pi/2,
yerr=sig_avg,

ls='',
color='cornflowerblu
fmt='o',

capsize=4,
elinewidth=1.2,
zorder=2,
label='OS data')

###################
x_smooth =
np.linspace(0.01,
180, 500)
y_fit, y_std =
GPfit(x=xi_mean *
180/np.pi,
y=rho_avg,
yerr=sig_avg,
x_output=x_smooth)
ax1.plot(x_smooth,
y_fit,
color='green',
lw=3, label='GP
fit', zorder=4,
```

```
alpha=.5)
ax1.fill_between(x_s

y_fit - y_std,

y_fit + y_std,

color='green',
alpha=0.1,
zorder=0,
label='GP
uncertainty')

###################
zeta =
np.linspace(0.01,
180, 100)
HD =
get_HD_curve(zeta
+ 1)
ax1.plot(zeta, HD,
ls='--',
label='Hellings-
Downs',
color='gray',
lw=3)


###################
ax1.axhline(0,
c='k', zorder=0,
alpha=.6)

###################
ax1.set_xlabel('Angl
of separation
(degrees)',
fontsize=16)
ax1.set_ylabel('Corr
fontsize=16)
ax1.grid(alpha=.2)


# ---- Right y-
axis: number of
pairs per bin ----
###################
ax2 = ax1.twinx()
ax2.bar(bin_centers
* 180/np.pi,

bin_counts,

edgecolor='black',

width=bin_widths *
180/np.pi,
        alpha=0.1,
color='gray',
zorder=1,
```

```
label='Number of
pairs per bin')

ax2.set_ylabel('Numb
of pairs',
fontsize=16)

if plotype==1:

ax2.set_ylim(0,
max(bin_counts) *
5)  # push
histogram to
bottom so it
doesn't crowd
correlations
elif plotype==2:

ax2.set_ylim(0,
max(bin_counts) *
1.5)  # push
histogram to
bottom so it
doesn't crowd
correlations

# ---- Legends
----
lines1, labels1 =
ax1.get_legend_handl
lines2, labels2 =
ax2.get_legend_handl
ax1.legend(lines1
+ lines2, labels1
+ labels2,
fontsize=13)

plt.tight_layout()

plt.show()
```

```
/opt/build_psrsoft/w
orkspace/micromamba/
envs/IPTA_Env/lib/py
thon3.11/site-packag
es/sklearn/gaussian_
process/kernels.py:4
45: ConvergenceWarni
ng: The optimal valu
e found for dimensio
n 0 of parameter k2_
_noise_level is clos
e to the specified l
ower bound 1e-05. De
creasing the bound a
nd calling fit again
may find a better va
lue.
```



Let us now plot it with Defiant tools

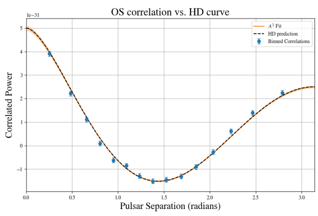
In [37…

```
ch, pars =
read_all_chains(ou
NRuns='auto')
os_obj =
OptimalStatistic(p
pta=pta,
chain=ch,
param_names=list(p
gwb_name='gw_curn_
orfs=['hd'])
```

In [38…

```
Nbins = 15

# Set our ORF to
HD
os_obj.set_orf(['h
hd_orf =
orf_functions.get_
xi_range =
np.linspace(0,np.p
[1:] # Define our
range of pulsar
separations
hd_mod =
hd_orf(xi_range)

# If params=None,
then DEFIANT will
use maximum
likelihood values
from
OS_obj.lfcore
#
xi,rho,sig,C,A2,A2
=
OS_obj.compute_OS(
output_OS =
os_obj.compute_OS(
A2_OS =
output_OS['A2']
A2s_OS =
output_OS['A2s']
idx_OS =
output_OS['idx']
xi_OS =
```

```
output_OS['xi']
rho_OS =
output_OS['rho']
sig_OS =
output_OS['sig']
C_OS =
output_OS['C']

# Plot the binned
pair correlation
plot.
f, ax =
defplot.create_cor
rho_OS, sig_OS,
C_OS, A2_OS,
A2s_OS,
bins=Nbins,
figsize=(10,6))
ax.plot(xi_range,1
k',label='HD
prediction')

ax.xaxis.label.set
ax.yaxis.label.set
plt.title('OS
correlation vs.
HD curve',
fontsize=20)

plt.legend()
#
plt.savefig("os.pn
# plt.close()
plt.show()
```



5.5 The Pair-Covariance OS (PCOS)

The standard OS treats pulsar pairs as **statistically independent**, which is an approximation: in reality, pairs sharing a pulsar are correlated through the common noise of that pulsar. The **Pair-Covariance OS (PC+OS)** accounts for these inter-pair correlations by computing the full covariance matrix $\mathbf{C}$ of the pair cross-correlations.

This improves the amplitude estimate when the GWB is strong relative to the noise, but at the cost of computing an $N_{\rm pairs} \times N_{\rm pairs}$ covariance matrix.

⏳ **This cell takes ~15 minutes.** It is included for completeness, feel free to run it after the tutorial. The standard OS and NMOS results are sufficient for the main analysis.

In [39…
```
"""
PCOS - Takes ~15
min !
"""
#print("DOING
PC+OS...")

#output_PCOS =
os_obj.compute_OS(
pair_covariance=Tr
```

Out[39]:
```
'\nPCOS - Takes ~
15 min !\n'
```

In [40…
```
"""
Nbins = 15

A2 =
```

```
output_PCOS['A2']
A2s =
output_PCOS['A2s']
idx =
output_PCOS['idx']
xi =
output_PCOS['xi']
rho =
output_PCOS['rho']
sig =
output_PCOS['sig']
C =
output_PCOS['C']

# Plot the binned
pair correlation
plot.
f, ax =
defplot.create_cor
rho_OS, sig_OS,
C_OS, A2_OS,
A2s_OS,
bins=Nbins,
figsize=(10,6))
ax.plot(xi_range,1
k',label='HD
prediction')

ax.xaxis.label.set
ax.yaxis.label.set
plt.title('OS
correlation vs.
HD curve',
fontsize=20)

plt.title('PC+OS')
plt.legend()
plt.show()
"""
```

Out[40]:
```
"\nNbins = 15\n\nA2 = output_PCOS['A2']\nA2s = output_PCOS['A2s']\nidx = output_PCOS['idx']\nxi = output_PCOS['xi']\nrho = output_PCOS['rho']\nsig = output_PCOS['sig']\nC = output_PCOS['C']\n\n# Plot the binned pair correlation plot.\nf, ax = defplot.create_correlation_plot(xi_OS, rho_OS, sig_OS, C_OS, A2_OS, A2s_OS, bins=Nbins, figsize=(10,6))\nax.plot(xi_range,10**(2*inj_params[f'gw_curn_pl_log10_A'])*hd_mod,'--k',label='HD prediction')\n\nax.xaxis.label.set_size(18)\nax.yaxis.label.set_size(18)\nplt.title('OS correlation vs. HD curve', fontsize=20)\n\nplt.title('PC+OS')\nplt.legend()\nplt.show()\n"
```

5.6 Measuring the significanc

We can also use the OS approach to estimate the significance of the GWB signal. Let us start with the signal-to-noise ratio.

5.6.1 The signal-to-noise ratio (S/N)

The OS signal-to-noise ratio is simply:

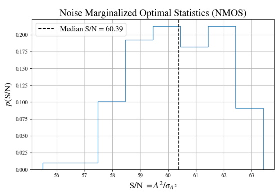
$$\hat{\rho} = \frac{\hat{A}^2}{\sigma(\hat{A}^2)}$$

A value $\hat{\rho} \gg 1$ indicates that the measured cross-correlations are inconsistent with noise alone.

Here we use the NMOS output to obtain a **distribution of $\hat{\rho}$ values, one per noise draw, whose median summarizes our detection significance.**

In [41…

```
A2 = output_NMOS['A2'
A2s = output_NMOS['A2s

# S/N per NM iteration, no uncertainty sampling needed
snr_nm = A2 / A2s
snr_nm_mean = np.median(snr_nm

plt.figure(figsi
5))
plt.hist(snr_nm, bins='auto', histtype='step', density=True)
plt.axvline(snr_
ls='--', color='k', label=f'Median S/N = {np.median(snr_r
plt.xlabel(r'S/N $= A^2 / \sigma_{A^2}$', fontsize=16)
plt.ylabel(r'$p(
N})$', fontsize=16)
plt.title('Noise Marginalized Optimal Statistics (NMOS)', fontsize=18)
plt.grid()
plt.legend(fonts
plt.show()
```



5.6.2 The p-value

From a S/N, we can compute a p-value, the probability of obtaining a S/N at least as large as observed, assuming the null hypothesis (no GWB) is true:

$$p = P(\hat{\rho} \geq \hat{\rho}_{\text{obs}}$$

The smaller the p-value, the higher the significance. The tricky part for PTAs is that we need to build a null hypothesis distribution empirically, since the GWB signal is always present in the data (i.e., it is present in the timing residuals of all pulsars). Two standard approaches are used to construct this null distribution:

> **Phase shifts**
>
> The timing residuals of each pulsar are phase-shifted in the Fourier domain by a random offset $\phi_a \in [0, 2\pi)$, independen for each pulsar.

This destroys the inter-pulsar correlations (and thus the HD signature) while preserving the spectral properties of the noise and any common red process. The OS is then recomputed on each shifted dataset, building a distribution of $\hat{\rho}$ under H_0.

Sky

scrambles

Instead of shifting phases, the sky positions of pulsars are randomly reassigned (or pulsars are swapped between each other), which changes the ORF values Γ_{ab} to effectively random values inconsistent with HD. This destroys the spatial correlation pattern while keeping the noise properties intact. The OS is recomputed for each scrambled configuratic again building the null $\hat{\rho}$ distribution

Both methods can be combined simultaneou applying phase shifts and sky scrambles together, to more efficiently decorrelate the data from

> any HD-like pattern.

The p-value is then estimated as:

$$p = \frac{\#\{\hat{\rho}_{\rm null} \geq}{N_{\rm null}}$$

where $\hat{\rho}_{\rm null}$ denotes S/N values from either phase-shifted, sky-scrambled (or both applied) datasets.

Building the null distributi with "phase shifts"

To convert the measured S/N into a **p-value**, we need to know the distribution of $\hat{\rho}$ under the null hypothesis H_0 (no GWB). Since the GWB signal is always present in simulated data, we cannot simply remove it, instead, we **destroy the HD spatial correlations** while preserving all spectral properties.

In a **phase shift**, the timing residuals of each pulsar are independentl shifted by a random phase $\phi_a \in [0, 2\pi)$ in the Fourier domain. This scrambles the relative phase between pulsars and erases any coherent inter-pulsar correlation, while keeping the single-pulsar noise properties unchanged.

> Note: we use fixed (injected) noise parameter here rather

> than marginaliz Noise marginaliz could be included by sampling params from the MCMC chain for each shift, but this increases the computatic cost significant

In [42…

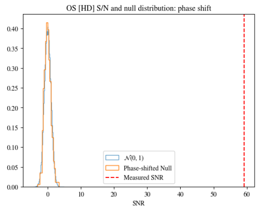
```
n_shifts = 500

# p_Phase (float): The p-value of the OS
# snr_Phase (float): The measured SNR of the OS
# n_dist_Phase (np.ndarray): The null distribution of the SNR
```

```
p_Phase,
snr_Phase,
n_dist_Phase
=
phase_shift_OS
params=inj_para
n_shifts=n_shi
print('Measured
P-
value:',p_Phase
limit:
P<=',1/
(len(n_dist_Pha
```

```
shifts: 100%|
███████████████
███████████████
███████████████
█| 500/500 [0
1:16<00:00,
6.52it/s]
Measured P-valu
e: 0.0 Upper li
mit: P<= 0.0019
9203187250996
```

In [43…

```
plt.title('OS
[HD] S/N and
null
distribution:
phase shift')

g =
np.random.randn
plt.hist(g,
bins=100,
histtype='step
alpha=0.7,
density
=True,
label="$
\mathcal{N}
(0,1)$")

plt.hist(n_dist
shifted
Null');
plt.axvline(sn
color='r',
linestyle='--'
SNR')
plt.xlabel('SNR
plt.legend()
plt.show()
```



Alternati null distributi The sky scramble

An alternative to phase shifts is the **sky scramble**: instead of shifting phases, the sky positions of pulsars are randomly reassigned. This changes the ORF values Γ_{ab} to effectively random values inconsistent with HD, while all timing properties remain intact.

Sky scrambles are particularly powerful because they test specifically the **angular pattern** of the correlations, any signal whose spatial signature is not HD-like should not survive scrambling.

> **Phase shifts vs. sky scrambles** Both methods are valid and

> complemen Phase shifts operate in the time/ frequency domain; sky scrambles operate in the angular domain. Using both (or combining them) gives a more robust null distributio The 2023 PTA papers used both approache to cross-check

> significanc
> estimates.
> These
> methods
> were
> highly
> discussed
> since
> 2023,
> but
> we
> won't
> go
> into
> the
> details
> in
> this
> tutorial.

In [44…

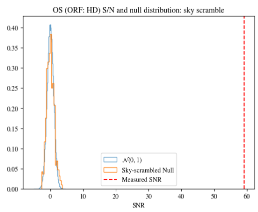
```
n_scrambles =
500

# p_Sky
(float): The
p-value of
the OS
# snr_Sky
(float): The
measured SNR
of the OS
# n_dist_Sky
(np.ndarray):
The null
distribution
of the SNR
p_Sky,
snr_Sky,
n_dist_Sky =
sky_scramble_OS
inj_params,
n_scrambles=n_s
swap_pos=False
print('Measured
P-
```

```
value:',p_Sky,
limit:
P<=',1/
(len(n_dist_Sky
```

```
scrambles: 10
0%|
███████████████
███████████████
█████████████|
500/500 [00:30<
00:00, 16.41it/
s]
Measured P-valu
e: 0.0 Upper li
mit: P<= 0.0019
9203187250996
```

In [45…

```
plt.title('OS
(ORF: HD) S/N
and null
distribution:
sky
scramble')

g =
np.random.randn
plt.hist(g,
bins=100,
histtype='step
alpha=0.7,
density
=True,
label="$
\mathcal{N}
(0,1)$")

plt.hist(n_dist
density=True,
histtype='step
scrambled
Null');
plt.axvline(sn
color='r',
linestyle='--'
SNR')
plt.xlabel('SNR
plt.legend()
plt.show()
```



5.6.3 The

equivale Gaussia significa

A common way to communicate a p-value is to convert it to an equivalent number of Gaussian standard deviations, the so-called **sigma (σ)** statistics. This is the value $\mathcal{N}_\sigma$ such that a one-sided Gaussian tail gives the same probability as the measured p-value. The higher the number of sigma, the more **incompatibl** the data are with the null hypothesis.

A key subtlety from PTAs is that the null distribution of $\hat{\rho}$ is **not Gaussian**. Previous discussions of the OS incorrectly assumed that the analytic null distribution of $\hat{\rho}$ is well-approximatec by a zero-mean unit-variance Gaussian. In reality, the null distribution has tails that differ significantly from a Gaussian, but which follows a **generalized chi-squared (GX2) distribution** i.e. a linear combination of chi-squared distributions. Assuming Gaussianity therefore gives the wrong p-value, and hence a misleading sigma. A correct assessment of the statistical significance requires fitting the null distribution with a GX2 model and deriving the p-value from this fit. The sigma value is then still reported as the Gaussian equivalent of that p-value, a convenient way to communicate significance in familiar units, even though the underlying distribution is not Gaussian.

The standard thresholds in GW astronomy are:

- $\mathcal{N}_\sigma \gtrsim 3$: **evidence** for a signal ($p \lesssim 1.3 \times$)
- $\mathcal{N}_\sigma \gtrsim 5$

: **detectio** ($p \lesssim 2.9 \times$)

The 2023 PTA papers (CPTA DR1, EPTA DR2, NANOGrav 15yr, PPTA DR3) reported HD correlations at the 2–4σ level depending on the dataset and method, motivating the use of "compelling evidence for" rather than "detection of" a GWB.

The GX2

null distribut

We now run both phase shifts and sky scrambles with $N = 500$ realizations each, and compare them to the analytic **generalized chi-squared (GX2)** null distribution.

The GX2 distribution arises because the OS statistic is a quadratic form in Gaussian random variables: its null distribution is a weighted sum of independent χ_1^2 variables, whose weights are the eigenvalues of the OS normalizatio matrix Q. This is computed analytically below using the **saddlepoin approximat** (Lugannani-Rice method), which is accurate in the tails where p-values are small.

> **Why the null distribut matters**

for sigma: Convertin a p-value to "$n\sigma$" implicitly assumes a Gaussian null distributio But for PTAs, the OS null distributio has non-Gaussian tails, assuming Gaussiani gives the wrong sigma. The correct procedure is to compute

> the p-value from the actual (GX2 or empirical null distributi then convert that p-value to a Gaussian-equivalen sigma $\mathcal{N}_\sigma = \Phi^{-1}$.

In [46…

```
# Compute with new Q matrix construction for the GX2. We won't go into the details here, we just place it here as it's long to compute
Q = build_Q_matri
inj_params)
eigvals_raw
=
np.linalg.eig
```

Let us start with phase shifts

In [47…

```
n_shifts =
500
p_Phase,
snr_Phase,
n_dist_Phase
=
phase_shift_
params=inj_p
n_shifts=n_s

print('Measu
P-value:',
p_Phase,'Upp
limit:
P<=',1/
(len(n_dist_
```

```
shifts: 10
0%|
████████████
████████████
████████████
██████████|
500/500 [0
1:16<00:00,
6.54it/s]
Measured P-v
alue: 0.0 Up
per limit: P
<= 0.0019920
3187250996
```

In [48…

```
# --- p-
values ---
n_phase =
len(n_dist_P
p_obs_phase,
_ =
gx2_survival
eigvals_raw)
p_obs_phase
=
p_obs_phase[
p_empirical_
=
np.mean(n_di
>=
snr_Phase)

p_obs_phase_
=
```

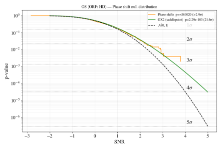
```
f"{p_obs_pha
({stats.norm
σ)"
p_phase_str
=
f"{p_empiric
({stats.norm
σ)" if
p_empirical_
> 0 else
f"<{1/
(n_phase+2):
(>{stats.nor
(n_phase+2))
σ)"
```

In [49…

```
plot_measure
= False
Nsig_max_plo
= 5 # try 5
or 23
GX2_SNRmax_p
= 5 # try 5
or 60
Gaussian_SNR
= 5 # try 5
or 60

############
fig, ax =
plt.subplots
6))

## Phase
shift null
distribution
n_dist_sorte
=
np.sort(n_di
survival_pha
= 1 -
np.arange(1,
n_phase +
1) /
(n_phase +
1)
ax.step(np.a
n_dist_sorte

np.append(1.
survival_pha

where='post'
label=f'Phas
shifts
p={p_phase_s
color='darkc

## GX2
x_gx2 =
```

```
np.linspace(
GX2_SNRmax_p
100)
survival_gx2
_ =
gx2_survival
eigvals_raw)
ax.plot(x_gx
survival_gx2
color='green
label=f'GX2
(saddlepoint
p={p_obs_pha

## Gaussian
x =
np.linspace(
Gaussian_SNR
1000)
ax.plot(x,
stats.norm.s
'k--',
label=r'$
\mathcal{N}
(0,1)$')

## Observed
SNR
if
plot_measure

ax.axvline(s
color='r',
linestyle='-
label=f'S/N
Phase =
{snr_Phase:.

## Sigma
levels
for ns in
range(1,
Nsig_max_plo
+ 1):
    p_level
=
stats.norm.s

ax.axhline(p
color='gray'
linestyle=':
lw=1)

ax.text(ax.g
[-2]-1,
p_level*1.2,
f' ${ns}\
\sigma$',
ha='left',
va='bottom',
```

```
fontsize=16)

ax.set_yscal
ax.set_xlabe
fontsize=16)
ax.set_ylabe
value',
fontsize=16)
ax.set_title
(ORF: HD) —
Phase shift
null
distribution
ax.tick_para
which='major
labelsize=14
ax.legend(fo
ax.grid(whic
alpha=0.2)
plt.tight_la
plt.show()
```



Now the sky scramb

In [50…

```
n_scrambles
= 500

os_obj.set_
p_Sky,
snr_Sky,
n_dist_Sky
=
sky_scrambl
n_scrambles
swap_pos=Fa

print('Meas
P-
value:',
p_Sky,'Uppe
limit:
P<=',1/
(len(n_dist
```

```
scrambles:
100%|
███████████
███████████
███████████
██████████|
500/500 [0
0:30<00:00,
16.14it/s]
Measured P-
value: 0.0
Upper limit
: P<= 0.001
99203187250
996
```

In [51…

```
#
Calculate
p-values
n_sky =
len(n_dist_
p_obs_sky,
_ =
gx2_surviva
eigvals_raw
p_obs_sky
=
p_obs_sky[0
p_empirical
=
np.mean(n_d
>=
snr_Sky)

p_obs_sky_s
=
f"{p_obs_sk
({stats.nor
σ)"
p_sky_str
=
f"={p_empir
({stats.nor
σ)" if
p_empirical
> 0 else
f"<{1/
(n_sky+2):.
(>{stats.no
(n_sky+2)):
σ)"
```

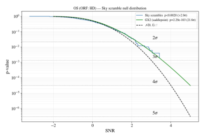
In [52…

```
plot_measur
= False
Nsig_max_pl
= 5 # try
5 or 23
GX2_SNRmax_
= 5 # try
```

```
5 or 60
Gaussian_SN
= 5 # try
5 or 60

##########
fig, ax =
plt.subplot
6))

## Sky
scramble
null
distributio
n_dist_sort
=
np.sort(n_d
survival_sk
= 1 -
np.arange(1
n_sky +
1) /
(n_sky +
1)
ax.step(np.
n_dist_sort

np.append(1
survival_sk

where='post
label=f'Sky
scrambles
p{p_sky_str
color='stee

## GX2
x_gx2 =
np.linspace
GX2_SNRmax_
100)
survival_gx
_ =
gx2_surviva
eigvals_raw
ax.plot(x_g
survival_gx
color='gree
label=f'GX2
(saddlepoin
p={p_obs_sk

##
Gaussian
x =
np.linspace
Gaussian_SN
1000)
ax.plot(x,
stats.norm.
```

```
'k--',
label=r'$
\mathcal{N}
(0,1)$')

##
Observed
SNR
if
plot_measu

ax.axvline(
color='dark
linestyle=
label=f'S/
N Sky =
{snr_Sky:.

## Sigma
levels
for ns in
range(1,
Nsig_max_pl
+ 1):

p_level =
stats.norm.

ax.axhline(
color='gray
linestyle=
lw=1)

ax.text(ax.
[-2]-1,
p_level*1.
f' ${ns}\
\sigma$',
ha='left',
va='bottom'
fontsize=16

ax.set_ysca
ax.set_xlab
fontsize=16
ax.set_ylab
value',
fontsize=16
ax.set_titl
(ORF: HD)
— Sky
scramble
null
distributio
ax.tick_par
which='majo
labelsize=1
ax.legend(f
ax.grid(whi
alpha=0.2)
```

```
plt.tight_l
plt.show()
```



Some selecte referer relatec to Sectior 5

- The Nanohe Gravita Wave Astronc (Taylor, 2021), very useful book about PTA. Interes again here.
- Optima strateg for gravita wave stochas backgr searche in pulsar timing arrays (Anholr

et al., 2022), basis Optima Statisti

- Noise-margin optimal statistic A robust hybrid frequen Bayesia statistic for the stochas gravita wave backgro in pulsar timing arrays (Vigela et al., 2018), noise margin Optima Statisti
- Analytic distribu of the optimal cross-correla statistic for

stochas gravita wave-backgr searche using pulsar timing arrays (Hazbo et al., 2023), referen for the GX2 null distribu

- Spatial and spectra charact of the gravita wave backgr with the PTA optimal statistic (Gersba et al., 2025), Per-frequer Optima Statisti relevan publica for defiant, good review on the optimal statistic method

6. Bon Sens as a Fun of Puls Num

How does the signi grow with the array

One of the most powerfu diagnost

of a PTA detectio is to ask: **does the significa grow as expecte when we add more pulsars**

For an ideal array with $N_{\rm psr}$ pulsars and $N_{\rm pairs} =$ pairs, the expectec S/N scales roughly as (Siemens et al. 2013):

$\hat{\rho} \propto \sqrt{N}$

Deviatio from this scaling can reveal:

- Pulsa with anon high noise
- A signa that is not perfe isotr
- Syste in the data

Our task

Using the precomp OS results, let us **rank the pulsars** by their contribu to the S/N and plot:

- $\hat{\rho}$ vs. $N_{\rm psr}$ (add pulsa one by one, rank by sens and comp with pred scali from Siem et al., 2013

In [53…

```
iterperp
= 50
Npsrlist
= [10,
20, 30,
40, 50,
60, 70,
80, 90,
100,
110,
120,
130]

SNRs =
[]
```

```
# Read
the
chain
for
CURN PL
ch,
pars =
read_all
NRuns='a

for
Npsrs
in
Npsrlist

print(f"
for
{Npsrs}
pulsars.

    #
The
same
process
is done
"iterper
times
for
each
pulsar
number

SNR_per_
= []
    for
i in
range(it

print(f"
iterperp
end="\r"


#
Create
pta
object
for
Npsrs
randomly
chosen
pulsars

selpsrs,
pta =
set_pta_
gwb_mode
Npsrs=Np
verbose=
```

```
#
Compute
the
Noise-
marginal
Optimal
Statisti
and
save
the
correspo
S/N

os_obj
=
OptimalS
pta=pta,
chain=ch
param_na
gwb_name
orfs=['h

output_N
=
os_obj.c
return_p

A2, A2s
=
(output_
for k
in
['A2','A

SNR_per_
/ A2s)


print("\
\n")

SNRs.app
```

```
Run for
10 pulsa
rs.
100.0%
OK.

Run for
20 pulsa
rs.
100.0%
OK.

Run for
30 pulsa
rs.
100.0%
OK.

Run for
40 pulsa
rs.
100.0%
OK.

Run for
50 pulsa
rs.
100.0%
OK.

Run for
60 pulsa
rs.
100.0%
OK.

Run for
70 pulsa
rs.
100.0%
OK.

Run for
80 pulsa
rs.
100.0%
OK.

Run for
90 pulsa
rs.
100.0%
OK.

Run for
100 puls
ars.
100.0%
OK.
```

```
Run for
110 puls
ars.
100.0%
OK.

Run for
120 puls
ars.
100.0%
OK.

Run for
130 puls
ars.
100.0%
OK.
```

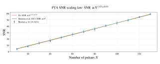
In [54…

```
#
Compute
median
and
spread
per
Npsr
SNRs_med
=
np.array
for
snr_list
in
SNRs])
SNRs_std
=
np.array
for
snr_list
in
SNRs])
SNRs_p16
=
np.array
16) for
snr_list
in
SNRs])
SNRs_p84
=
np.array
84) for
snr_list
in
SNRs])

Npsrlist
=
np.array

########
```

```
### Fit
for a
power-
law
model
########

#
Define
the
power-
law
model
def
power_la
alpha,
C):

return
C *
N**alpha

# Fit
in
linear
space
(robust
with
median)
popt,
pcov =
curve_fi
Npsrlist
SNRs_med
p0=[0.5,
1.0])
alpha_fi
C_fit =
popt
alpha_er
C_err =
np.sqrt(
N_fine
=
np.linsp
Npsrlist
300)
f_fit
=
power_la
*popt)

########
###
Same
but
with
predicte
scaling
relation
```

```
from
Siemens
et al.
2013,
alpha=1.
Here C
is
anchored
to the
data
########

power_la
=
lambda
N, C: C
*
N**1.0
popt_th,
_ =
curve_fi
Npsrlist
SNRs_med
C_theory
=
popt_th[

###
Plot
plt.figu
5))

plt.erro
SNRs_med
yerr=[SN
-
SNRs_p16
SNRs_p84
-
SNRs_med
fmt='o',
color='s
capsize=
label='M
± 1σ
(16–
84%)')
plt.plot
f_fit,
'r--',
linewidt
label=fr
SNR $
\propto
N^{{{alp
\pm
{alpha_e
plt.plot
C_theory
*
```

```
N_fine**
'g-',
linewidt
label=r'
et al.
2013:
SNR $
\propto
N^{1}$')

plt.xlab
of
pulsars
$N$',
fontsize
plt.ylab
fontsize
plt.xtic
plt.ytic
plt.lege
plt.grid
which='b
alpha=0.

plt.supt
SNR
scaling
law:
SNR $
\propto
N^{{{alp
\pm
{alpha_e
fontsize
plt.tigh
plt.show
```



Some selec refer relate to Secti 6

- The Nanc Grav Wave Astro

(Tayl 2021 very usefu book abou PTA, why not addir it ?

- The stoch backg scalir laws and time to detec for pulsa timin array (Sien et al., 2013 Scali laws for the GWB signi in PTAs

In [ ]:

In [ ]:

In [ ]:

In [ ]: