PTAs and the gravitational wave background

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EPTA

INTERNATIONAL PULSAR TIMING ARRAY IPTA

erc

(Sesana & Figueroa “Nanohertz Gravitational Waves”, arXiv:2512.18822)

Pulsar response to an incoming GW

The GW induces a Doppler shift in the ToA of the pulses. As studied In the context of spacecraft ranging (Estabrook 1975)

$$z(t) = \frac{\nu(t) - \nu_0}{\nu_0} = \frac{\delta\nu(t)}{\nu_0}$$

$$z(t) = \frac{1}{2} p_i p_j \int_{t_p}^{t} dt' \frac{\partial}{\partial t'} h_{ij}[t', (t - t_p)\hat{p}]$$

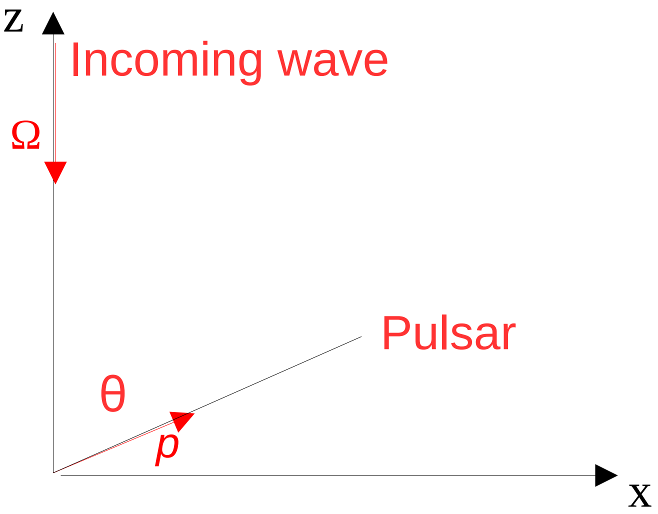
For the system on the right this becomes

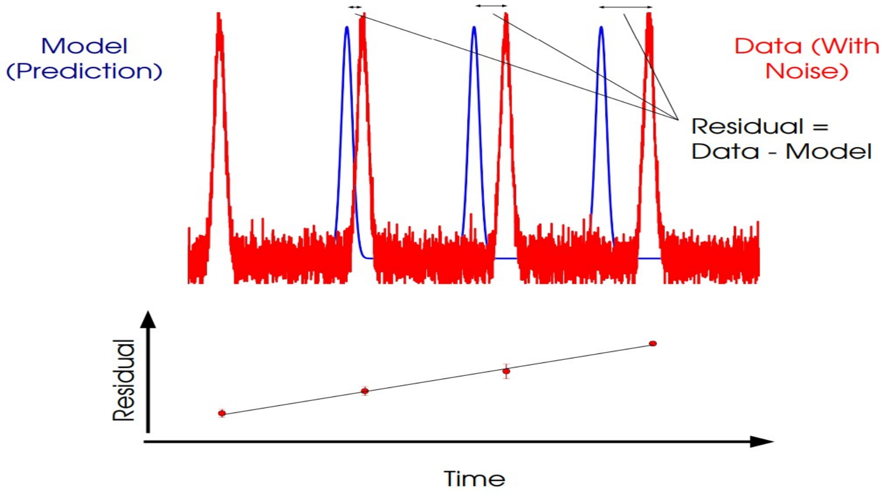
$$z(t) = \frac{1}{2}(1 - \cos\theta)[h_{xx}(t_p) - h_{xx}(t)]$$

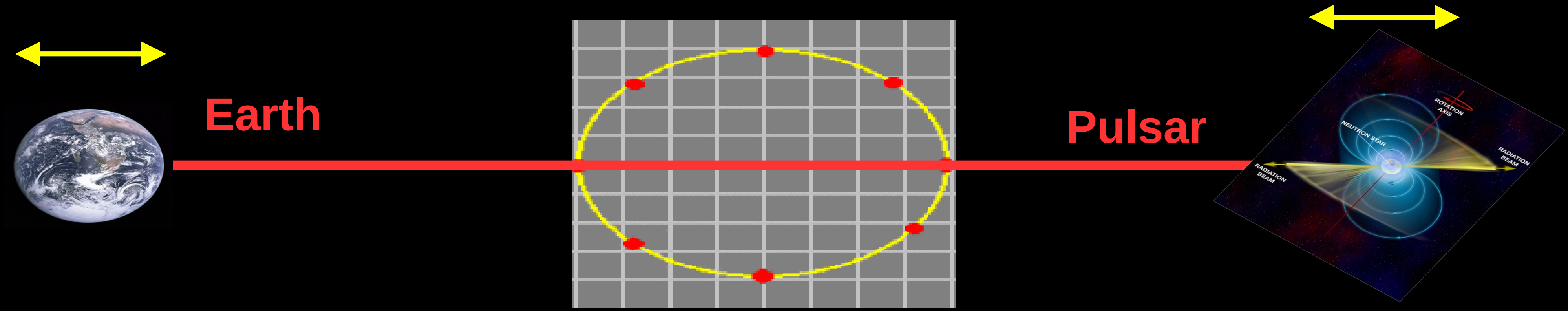
Where $t - t_p \equiv \tau = (L/c)(1 + \hat{\Omega}\cdot\hat{p})$

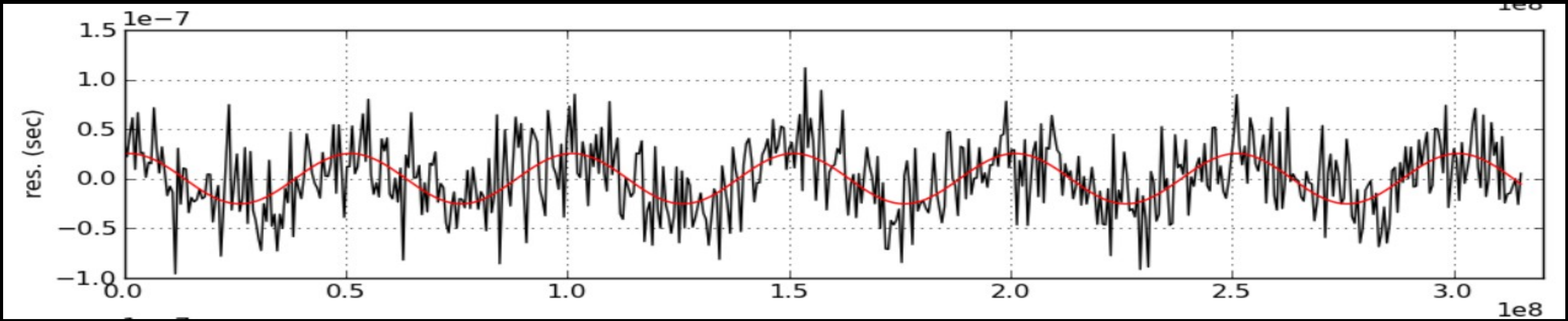
What PTA measures is the integral of this dephasing, which we call the 'residual'

$$r(t) = \int_0^t dt' z(t', \hat{\Omega})$$






Earth
Pulsar
ROTATION AXIS
NEUTRON STAR
RADIATION BEAM
RADIATION BEAM


1e−7
1.5
1.0
0.5
0.0
−0.5
−1.0
res. (sec)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1e8

Generalization for generic GW and pulsar directions

$$\hat{\Omega} = -(\sin\theta\cos\phi)\,\hat{x} - (\sin\theta\sin\phi)\,\hat{y} - \cos\theta\,\hat{z}$$
$$\hat{p} = (\sin\theta_p\cos\phi_p)\,\hat{x} + (\sin\theta_p\sin\phi_p)\,\hat{y} + \cos\theta_p\,\hat{z}.$$

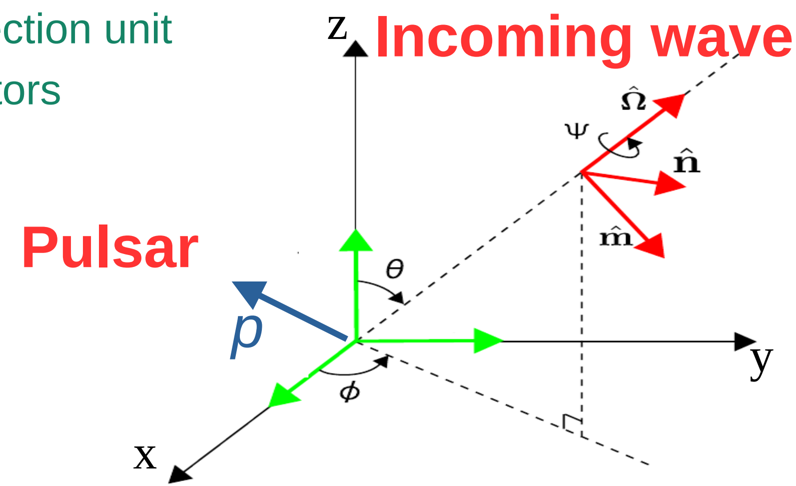
Direction unit vectors

$$h_{ij}(t,\hat{\Omega}) = e^{+}_{ij}(\hat{\Omega})h_{+}(t,\hat{\Omega}) + e^{\times}_{ij}(\hat{\Omega})h_{\times}(t,\hat{\Omega})$$

Strain

$$e^{+}_{ij}(\hat{\Omega}) = \hat{m}_i\hat{m}_j - \hat{n}_i\hat{n}_j\,,$$
$$e^{\times}_{ij}(\hat{\Omega}) = \hat{m}_i\hat{n}_j + \hat{n}_i\hat{m}_j\,.$$

Polarization tensors



$$\hat{m} = (\sin\phi\cos\psi - \sin\psi\cos\phi\cos\theta)\hat{x} - (\cos\phi\cos\psi + \sin\psi\sin\phi\cos\theta)\hat{y} + (\sin\psi\sin\theta)\hat{z}\,,$$
$$\hat{n} = (-\sin\phi\sin\psi - \cos\psi\cos\phi\cos\theta)\hat{x} + (\cos\phi\sin\psi - \cos\psi\sin\phi\cos\theta)\hat{y} + (\cos\psi\sin\theta)\hat{z}.$$

Redshift

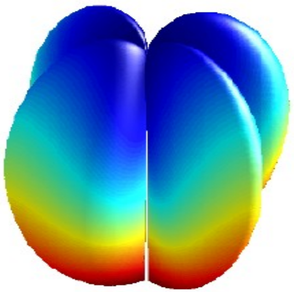
$$\boxed{z(t,\hat{\Omega}) = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{p}^i\hat{\Omega}_i}\left\{e^{+}_{ij}(\hat{\Omega})\left[h_{+}(t_p,\hat{\Omega}) - h_{+}(t,\hat{\Omega})\right] - e^{\times}_{ij}(\hat{\Omega})\left[h_{\times}(t_p,\hat{\Omega}) - h_{\times}(t,\hat{\Omega})\right]\right\}}$$

which can be written in compact form as

$$z(t,\hat{\Omega}) = \sum_A F^A(\hat{\Omega})[h_A(t_p,\hat{\Omega}) - h_A(t,\hat{\Omega})]$$

where

$$F^{+}(\hat{\Omega}) = \frac{1}{2}\frac{(\hat{m}\cdot\hat{p})^2 - (\hat{n}\cdot\hat{p})^2}{1+\hat{\Omega}\cdot\hat{p}}$$
$$F^{\times}(\hat{\Omega}) = \frac{(\hat{m}\cdot\hat{p})\,(\hat{n}\cdot\hat{p})}{1+\hat{\Omega}\cdot\hat{p}}\,.$$



We start by writing the GWB as a superposition of plane waves in the Fourier domain

$$h_{ij}(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} \mathrm{d}f \int_{4\pi} \mathrm{d}\hat{\Omega}\, \tilde{h}(f,\hat{\Omega}) e^{-2\pi i f t} e_{ij}^{A}(\hat{\Omega})$$

The recall the form of the residual

$$z(t,\hat{\Omega}) = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{p}^i\hat{\Omega}_i}\left\{e_{ij}^{+}(\hat{\Omega})\left[h_+(t_p,\hat{\Omega}) - h_+(t,\hat{\Omega})\right] - e_{ij}^{\times}(\hat{\Omega})\left[h_\times(t_p,\hat{\Omega}) - h_\times(t,\hat{\Omega})\right]\right\}$$

It is then straightforward to write the GWB induced residual as

$$z(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2\hat{\Omega}\, F^A(\hat{\Omega})\tilde{h}_A(f,\hat{\Omega}) e^{-2\pi i f t}\left[1 - e^{-2\pi i f \tau}\right]$$

We make the assumption of a stationary, isotropic and unpolarized GWB

$$\langle \tilde{h}_A^*(f,\hat{\Omega})\tilde{h}'_A(fi,\hat{\Omega}')\rangle = \delta(f-f')\frac{\delta^2(\hat{\Omega},\hat{\Omega}')}{4\pi}\delta_{AA'}\frac{1}{2}S_h(f)$$

$\boldsymbol{S_h}$ here is the power spectral density of the GWB, which describes how much power per unit frequency is carried by the GWB

We now want to compute the expectation value of the correlation between the redshift of two pulsars *a* and *b* in the sky

$$z_a(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2\hat{\Omega}\, F_a^A(\hat{\Omega})\, \tilde{h}_A(f,\hat{\Omega})\, e^{-2\pi i f t}\left[1 - e^{-2\pi i f \tau_a}\right]$$

$$z_b(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2\hat{\Omega}\, F_b^A(\hat{\Omega})\, \tilde{h}_A(f,\hat{\Omega})\, e^{-2\pi i f t}\left[1 - e^{-2\pi i f \tau_b}\right]$$

$$\langle z_a(t) z_b(t)\rangle = \sum_{AA'} \int df\, df' \int d^2\hat{\Omega}\, d^2\hat{\Omega}' F_a^A(\hat{\Omega}) F_b^{A'}(\hat{\Omega}') e^{-2\pi i (f+f')t} \left\langle \tilde{h}_A(f,\hat{\Omega}) \tilde{h}_{A'}(f',\hat{\Omega}')\right\rangle \left(1 - e^{-2\pi i f \tau_a}\right)\left(1 - e^{-2\pi i f' \tau_b}\right)$$

The correlation is made of four terms

$$\langle z_a z_b\rangle = \langle z_a z_b\rangle_{EE} + \langle z_a z_b\rangle_{PE} + \langle z_a z_b\rangle_{EP} + \langle z_a z_b\rangle_{PP}$$

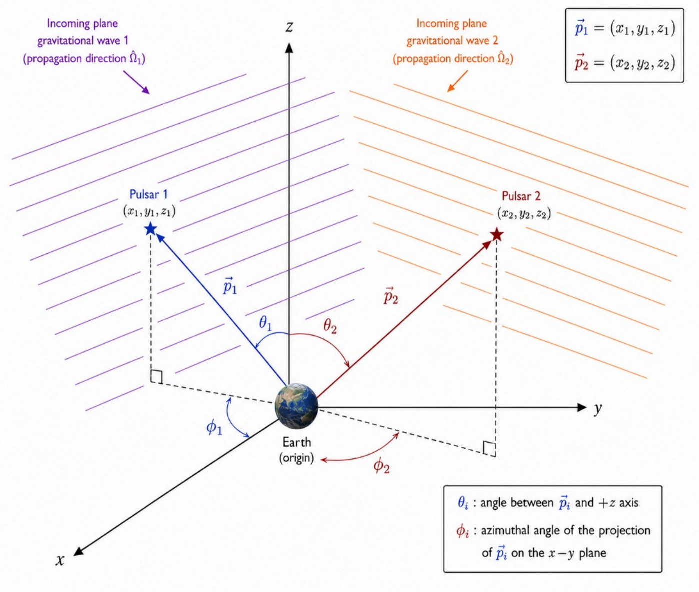
Where

$$\langle z_a z_b\rangle_{EE} = \sum_{AA'} \int df\, df' \int d^2\hat{\Omega}\, d^2\hat{\Omega}'\, F_a^A F_b^{A'}\, e^{-2\pi i (f+f')t} \left\langle \tilde{h}_A(f,\hat{\Omega}) \tilde{h}_{A'}(f',\hat{\Omega}')\right\rangle,$$

$$\langle z_a z_b\rangle_{PE} = -\sum_{AA'} \int df\, df' \int d^2\hat{\Omega}\, d^2\hat{\Omega}'\, F_a^A F_b^{A'}\, e^{-2\pi i (f+f')t}\, e^{-2\pi i f \tau_a} \left\langle \tilde{h}_A(f,\hat{\Omega}) \tilde{h}_{A'}(f',\hat{\Omega}')\right\rangle,$$

$$\langle z_a z_b\rangle_{EP} = -\sum_{AA'} \int df\, df' \int d^2\hat{\Omega}\, d^2\hat{\Omega}'\, F_a^A F_b^{A'}\, e^{-2\pi i (f+f')t}\, e^{-2\pi i f' \tau_b} \left\langle \tilde{h}_A(f,\hat{\Omega}) \tilde{h}_{A'}(f',\hat{\Omega}')\right\rangle,$$

$$\langle z_a z_b\rangle_{PP} = \sum_{AA'} \int df\, df' \int d^2\hat{\Omega}\, d^2\hat{\Omega}'\, F_a^A F_b^{A'}\, e^{-2\pi i (f+f')t}\, e^{-2\pi i (f\tau_a + f'\tau_b)} \left\langle \tilde{h}_A(f,\hat{\Omega}) \tilde{h}_{A'}(f',\hat{\Omega}')\right\rangle$$



Now recall that the tau has form $t - t_p \equiv \tau = (L/c)(1 + \hat{\Omega}\cdot\hat{p})$

and *L* is kpc such that *fL/c >> 1*

Only the *EE* term survives

$$\langle z_a(t) z_b(t)\rangle = \frac{1}{2}\int_{-\infty}^{\infty} df\, S_h(f) \int d^2\hat{\Omega} \frac{1}{4\pi} \sum_{A=+,\times} F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}).$$

The integral of the pattern functions is cumbersome but doable:

$$C(\zeta_{ab}) \equiv \sum_{A=+,\times} \int \frac{\mathrm{d}^2\hat{\Omega}}{4\pi} F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) = \frac{1}{4}\left[1 + \frac{\cos\zeta_{ab}}{3} + 4(1-\cos\zeta_{ab})\ln\left(\sin\frac{\zeta_{ab}}{2}\right)\right]$$

We are not done yet! We observe residuals $r(t) = \int_0^t dt' z(t', \hat{\Omega})$.

We therefore want the expectation value $r_{ab} = \langle r_a(t) r_b(t)\rangle$

But *t* is only in the exponential $z(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2\hat{\Omega}\, F^A(\hat{\Omega}) \tilde{h}_A(f,\hat{\Omega}) e^{-2\pi i f t} \left[1 - e^{-2\pi i f \tau}\right]$

So we just have $r_{ab} = C(\zeta_{ab}) \int_0^{\infty} df \frac{S_h(f)}{4\pi^2 f^2}$

We define $S_h(f) = \frac{2}{3} P_h(f)$

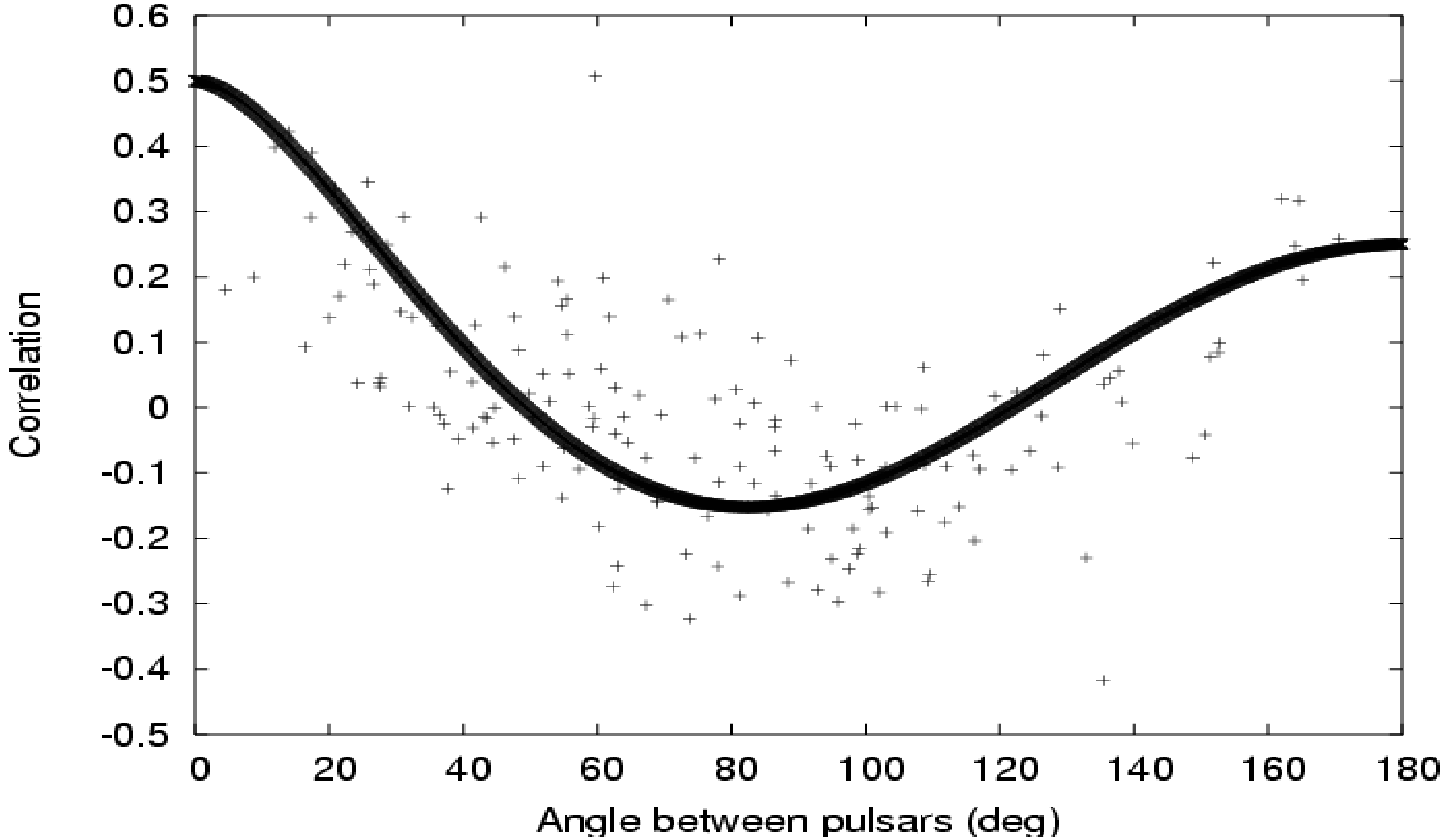
$$\Gamma(\zeta_{ab}) = \frac{3}{2} C(\zeta_{ab})(1 + \delta_{ab})$$

..to finally get

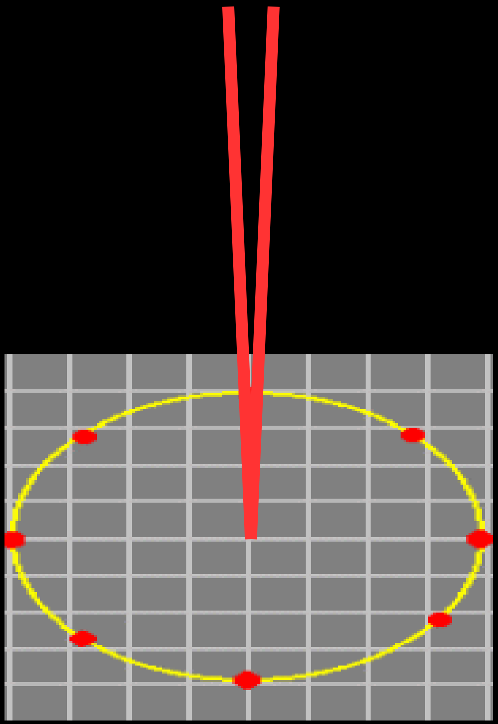
$$\boxed{r_{ab} = \Gamma(\zeta_{ab}) \int_0^{\infty} df\, P_h(f)}$$

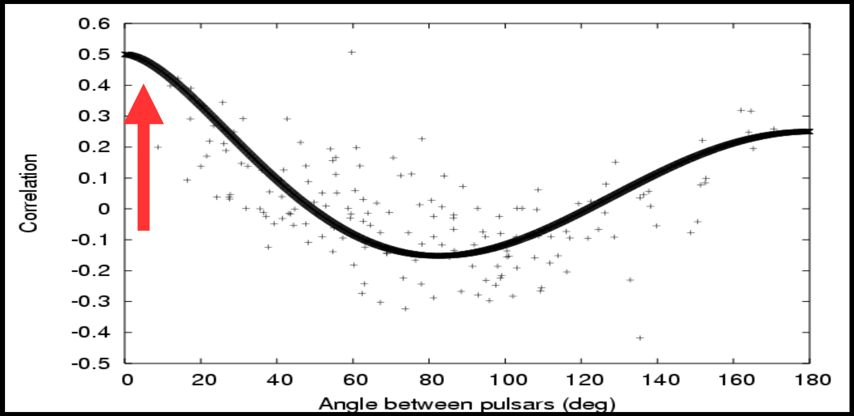
$\Gamma(\zeta_{ab})$ Depends on the relative angle between pulsars a and b only and it is the famous HD curve (HD 1983)

$$\Gamma(\zeta_{ab}) = \frac{3}{2}C(\zeta_{ab})(1+\delta_{ab}) \qquad C(\zeta_{ab}) = \frac{1}{4}\left[1 + \frac{\cos\zeta_{ab}}{3} + 4(1-\cos\zeta_{ab})\ln\left(\sin\frac{\zeta_{ab}}{2}\right)\right]$$

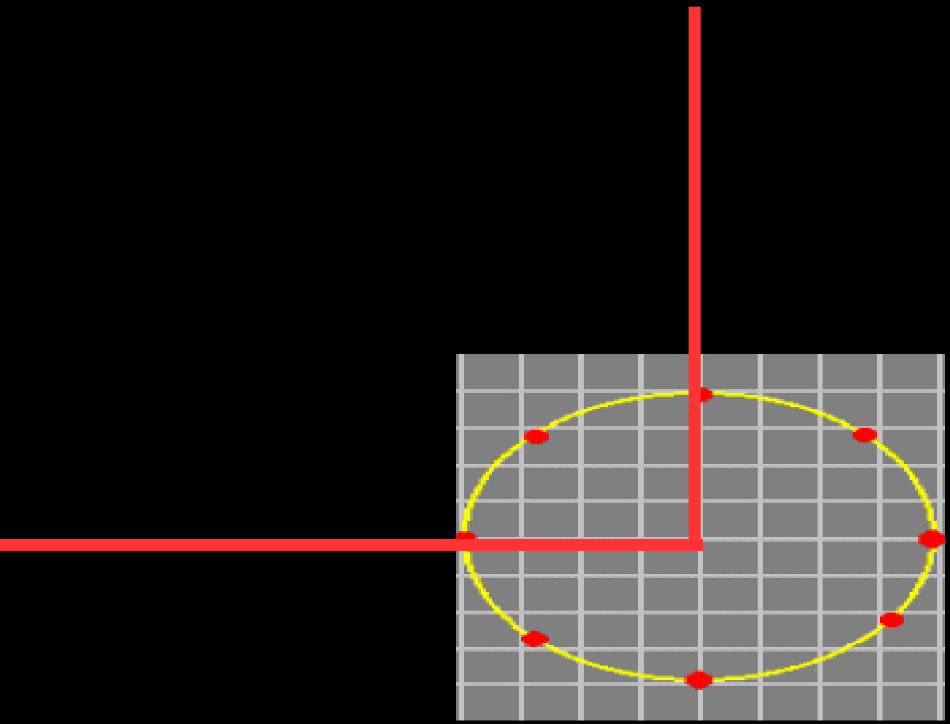


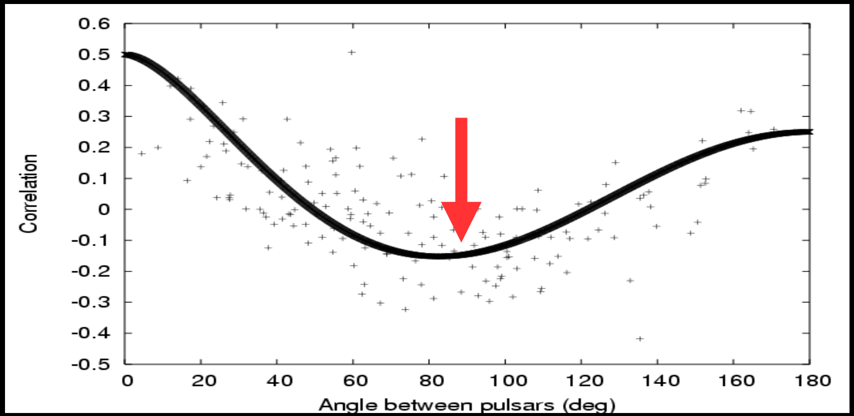
(Hellings & Downs 1983)





Correlation
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
20
40
60
80
100
120
140
160
180
Angle between pulsars (deg)




0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
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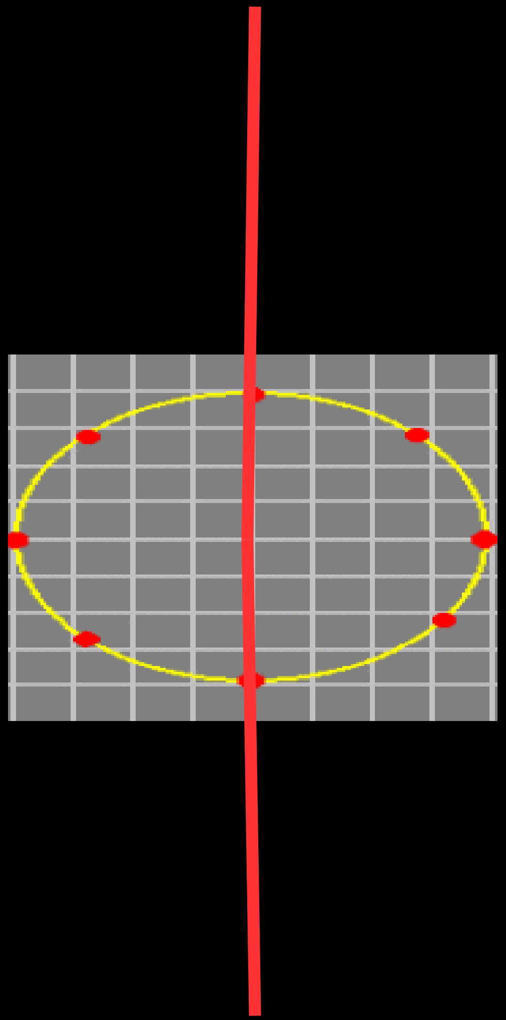
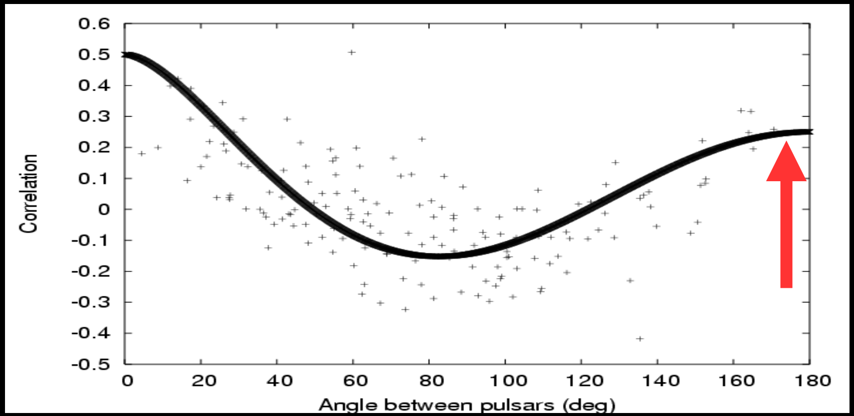
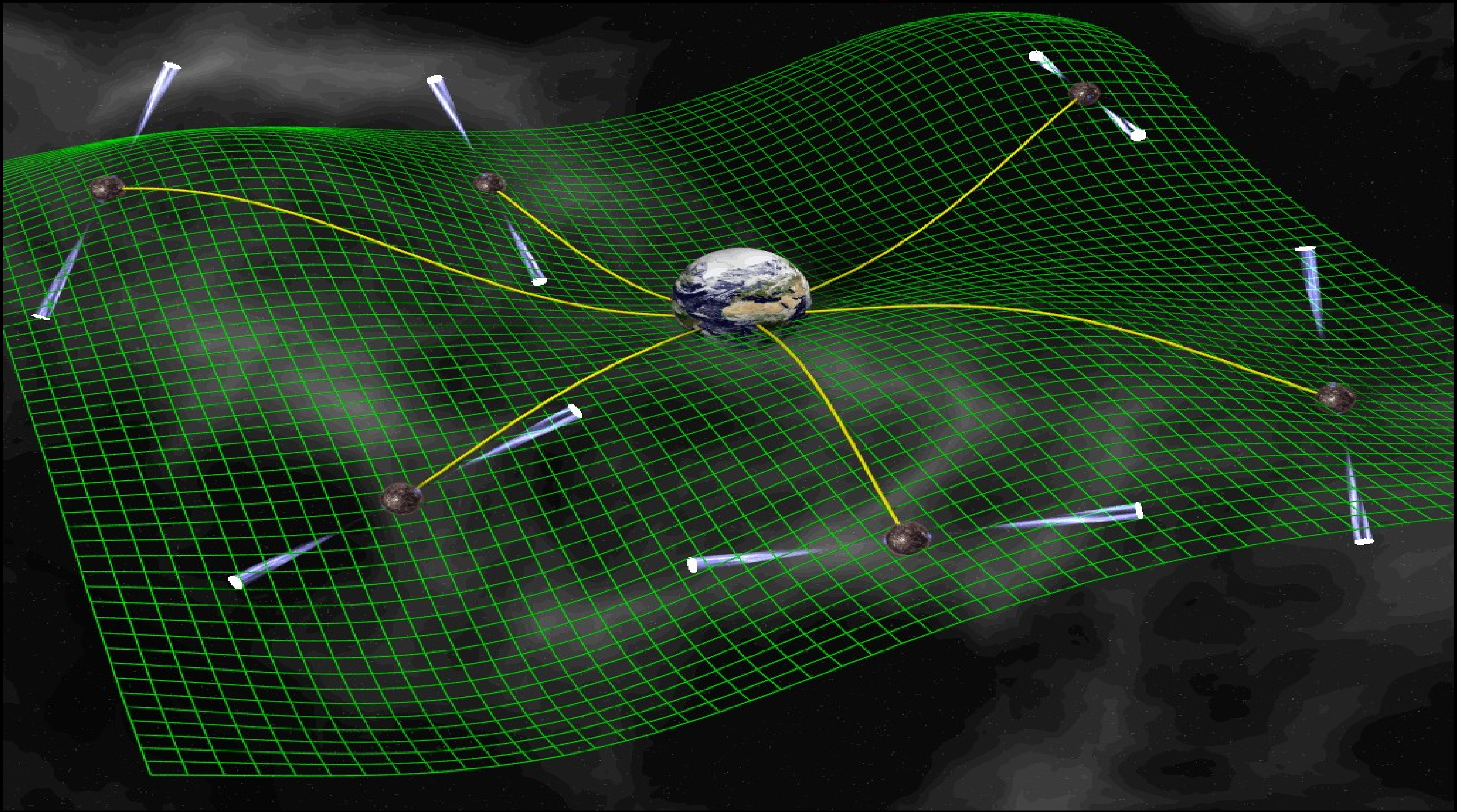
Correlation
Angle between pulsars (deg)





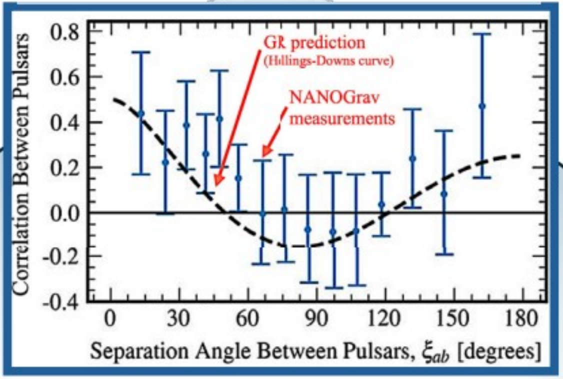
Correlation
Angle between pulsars (deg)
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
20
40
60
80
100
120
140
160
180

Here comes the concept of PTA



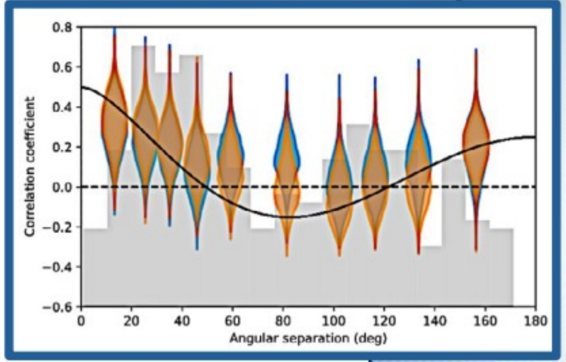
From Taylor 2025

Agazie *et al* 2023 *ApJL* 951 L8



NANOGrav

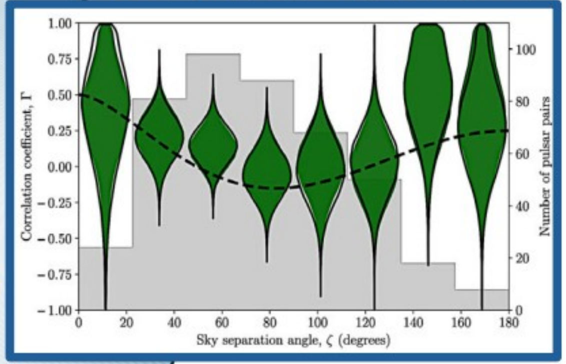
Antoniadis et al 2023 A&A 678, A50



EPTA

InPTA
Indian Pulsar Timing Array

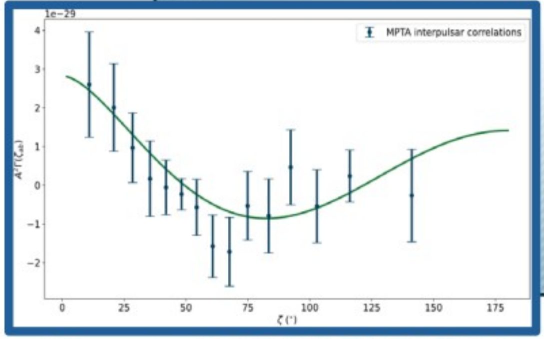
Reardon *et al* 2023 *ApJL* 951 L6



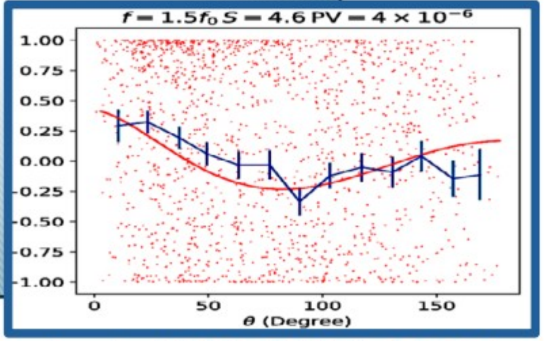
PPTA

MPTA

CPTA



Miles et al 2025 MNRAS 536 2



Xu et al 2023 RAA 23 075024

Ok for the correlation, but what's the amplitude and shape of signal we expect to detect?

We start again from our wave superposition

$$h_{ij}(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} \mathrm{d}f \int_{4\pi} \mathrm{d}\hat{\Omega}\, \tilde{h}(f,\hat{\Omega}) e^{-2\pi i f t} e^{A}_{ij}(\hat{\Omega})$$

$$\langle \tilde{h}^*_A(f,\hat{\Omega}) \tilde{h}'_A(fi,\hat{\Omega}') \rangle = \delta(f-f') \frac{\delta^2(\Omega,\Omega')}{4\pi} \delta_{AA'} \frac{1}{2} S_h(f)$$

$$\langle h_{ij}(t)\, h_{ij}(t) \rangle = 4 \int_0^{\infty} \mathrm{d}f\, S_h(f) \equiv 2 \int_0^{\infty} \mathrm{d}\ln f\, h_c^2(f)$$

Here we have defined the characteristic strain

$$\boxed{h_c^2(f) = 2fS(f)}$$

A GWB is generally described in terms of the energy density it carries which, from GR is

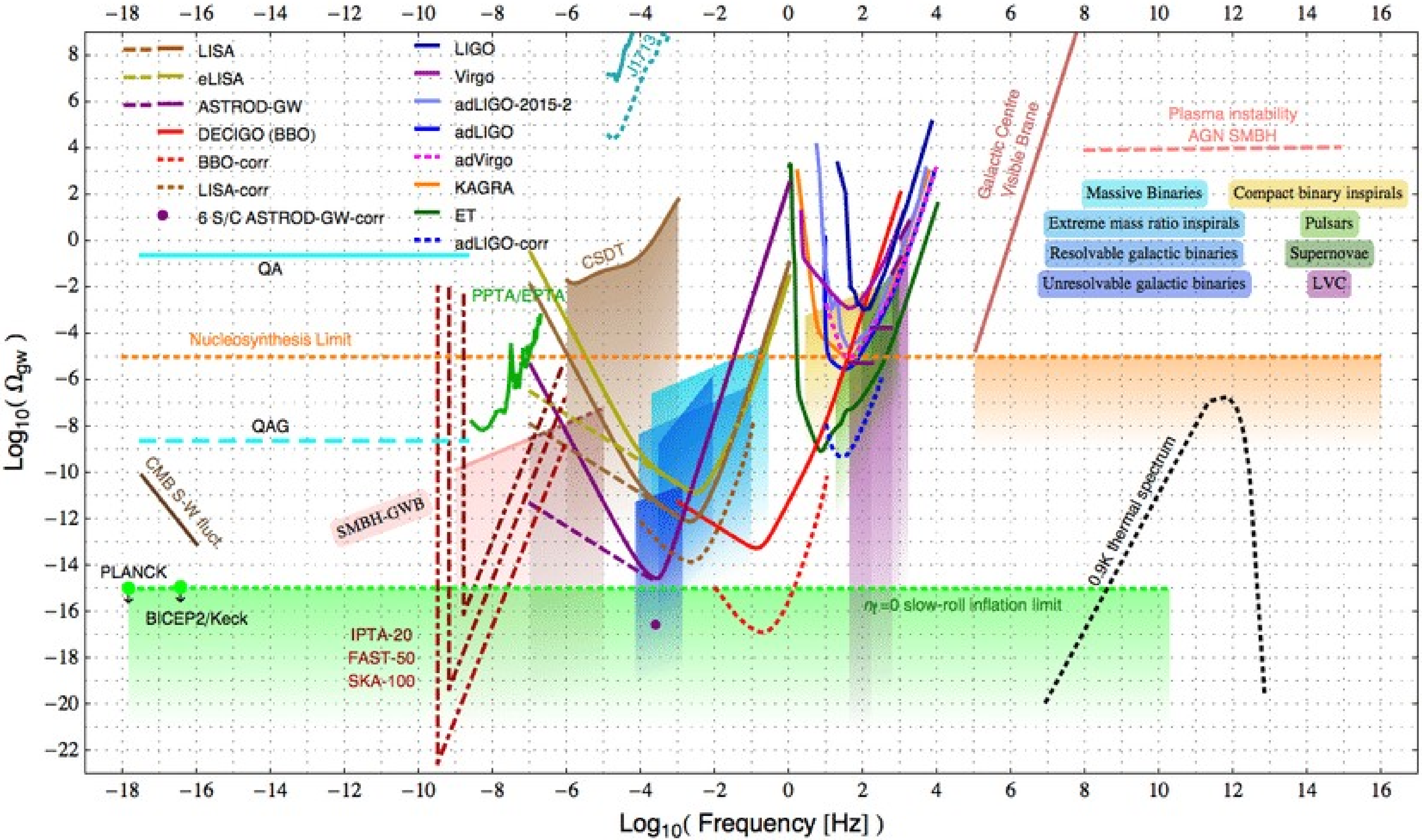
$$\epsilon_{\mathrm{GW}} \equiv \rho_{\mathrm{GW}} c^2 = T_{00} = \frac{c^2}{32\pi G} \langle \dot{h}_{ij}(t)\, \dot{h}_{ij}(t) \rangle$$

Since t is only in the exponent of h, it is straightforward to demonstrate

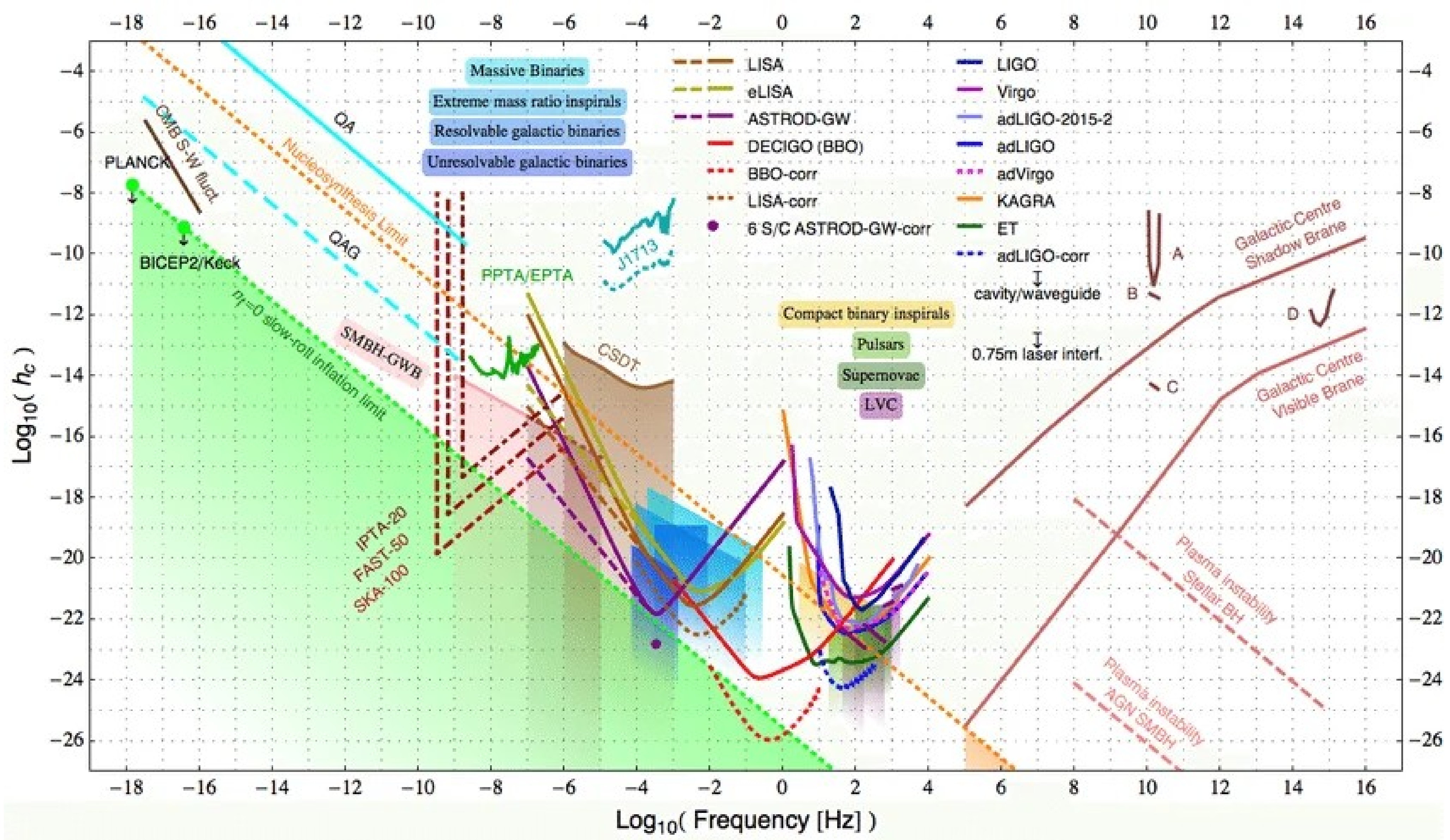
$$\epsilon_{\mathrm{GW}} \equiv c^2 \int_0^{\infty} \mathrm{d}\ln f \left(\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln f} \right) = \frac{\pi c^2}{4G} \int_0^{\infty} \mathrm{d}\ln f\, f^2 h_c^2(f) = \frac{\pi c^2}{2G} \int_0^{\infty} \mathrm{d}\ln f\, f^3 S_h(f)$$

We can now connect all the relevant quantities used in literature to describe the amplitude and shape of a GWB

$$\boxed{\Omega_{\mathrm{GW}}(f) \equiv \frac{1}{\rho_c} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln f} \equiv \frac{8\pi G}{3H_0^2} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \frac{4\pi^2}{3H_0^2} f^3 S_h(f)}$$



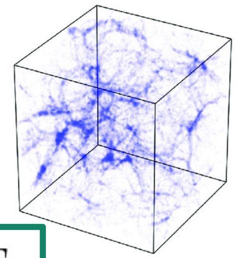
LISA
eLISA
ASTROD-GW
DECIGO (BBO)
BBO-corr
LISA-corr
6 S/C ASTROD-GW-corr
LIGO
Virgo
adLIGO-2015-2
adLIGO
adVirgo
KAGRA
ET
adLIGO-corr
QA
QAG
CSDT
PPTA/EPTA
Nucleosynthesis Limit
CMB S-W fluct
SMBH-GWB
PLANCK
BICEP2/Keck
IPTA-20
FAST-50
SKA-100
J1713
Galactic Centre
Visible Brane
Plasma instability
AGN SMBH
Massive Binaries
Compact binary inspirals
Extreme mass ratio inspirals
Pulsars
Resolvable galactic binaries
Supernovae
Unresolvable galactic binaries
LVC
n_t=0 slow-roll inflation limit
0.9K thermal spectrum
$\mathrm{Log}_{10}(\Omega_{gw})$
Log_{10}(Frequency [Hz])



Log10(Frequency [Hz])
Log10(hc)
LISA
eLISA
ASTROD-GW
DECIGO (BBO)
BBO-corr
LISA-corr
6 S/C ASTROD-GW-corr
LIGO
Virgo
adLIGO-2015-2
adLIGO
adVirgo
KAGRA
ET
adLIGO-corr
Massive Binaries
Extreme mass ratio inspirals
Resolvable galactic binaries
Unresolvable galactic binaries
Compact binary inspirals
Pulsars
Supernovae
LVC
QA
QAG
CMB S-W fluct
PLANCK
BICEP2/Keck
Nucleosynthesis Limit
nt=0 slow-roll inflation limit
SMBH-GWB
PPTA/EPTA
J1713
CSDT
IPTA-20
FAST-50
SKA-100
cavity/waveguide
0.75m laser interf.
A
B
C
D
Galactic Centre Shadow Brane
Galactic Centre Visible Brane
Plasma instability Stellar BH
Plasma instability AGN SMBH

For an astrophysical population of sources characterized by a given number density and emitting a given energy spectrum we can write

$$\epsilon_{\rm GW} = \int d\ln f \int dz \int d\mathcal{M} \frac{1}{1+z} \frac{d^2 n}{dz\, d\mathcal{M}} \frac{dE}{d\ln f_{\rm r}}$$



Equating the two expression for the energy density we get

$$\boxed{h_c^2(f) = \frac{4G}{\pi c^4} \frac{1}{f^2} \int dz \int d\mathcal{M} \frac{d^2 n}{dz\, d\mathcal{M}} \frac{dE}{d\ln f_{\rm r}}}$$

For a circular GW driven binary

$$\frac{dE}{df_{\rm r}} = \frac{dE}{dt_{\rm r}} \frac{dt_{\rm r}}{df_{\rm r}} = \frac{\pi}{3G} \frac{(G\mathcal{M})^{5/3}}{(\pi f_{\rm r})^{1/3}}$$

And we can write

$$h_c^2(f) = \frac{4(G\mathcal{M}_0)^{5/3}}{3\pi^{1/3} c^2} \frac{1}{f^{4/3}} n_0 \langle (1+z)^{-1/3} \rangle$$

where n_o is the present day GW source remnant number density

The PTA signal si dominated by very massive binaries that reside in giant ellipticals (mostly BCGs).

-BCG number density today is ~10^{-4}Mpc^{-3}

-Each BCG had 1 major merger in the past 10Gyr → each BCG is a MBHB remnant and n_o~10^{-4}Mpc^{-3}

$$\boxed{h_c(f) \approx 10^{-15} \left(\frac{f}{1\ \mathrm{year}^{-1}}\right)^{-2/3} \left(\frac{\mathcal{M}}{10^9\ \mathrm{M}_\odot}\right)^{5/6} \left(\frac{n_0}{10^{-4}\ \mathrm{Mpc}^{-3}}\right)^{1/2} \chi(z)^{1/2}.}$$

So we expect a GWB of A~10^{-15} with a -2/3 spectral slope

But the actual GWB buildup is different

Izquierdo-Villalba+ 2023

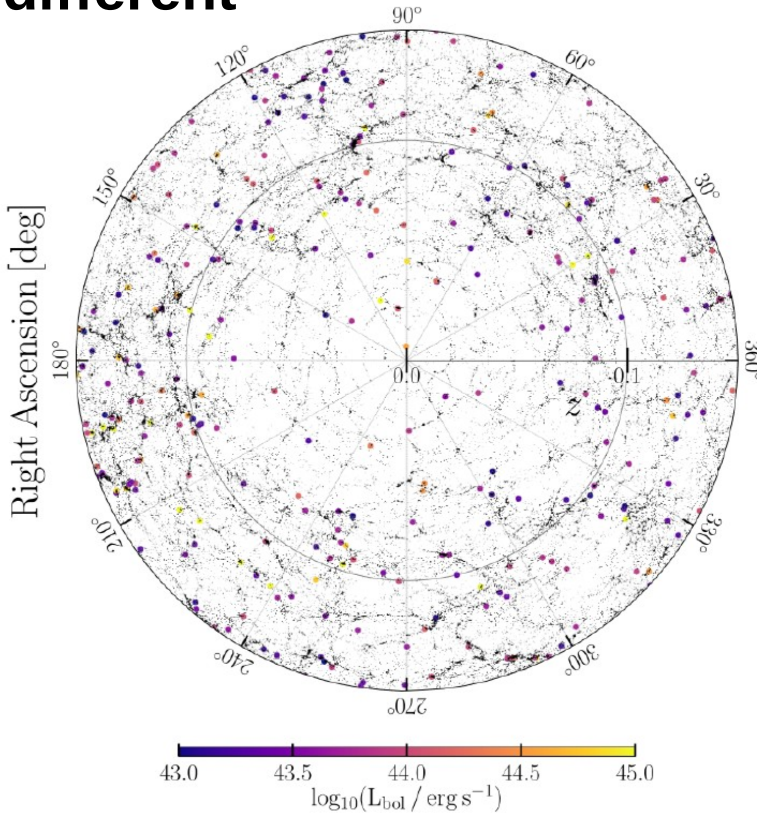
$$h_c^2(f) = \frac{4G}{\pi c^4}\frac{1}{f^2}\int \mathrm{d}z \int \mathrm{d}\mathcal{M}\frac{\mathrm{d}^2 n}{\mathrm{d}z\,\mathrm{d}\mathcal{M}}\frac{\mathrm{d}E}{\mathrm{d}\ln f_\mathrm{r}}$$

is NOT how the nano-Hz GWB build-up actually works

Our PTA is perturbed by a number of discrete MBHBs with different masses at different *z*, each emitting a monochromatic wave

To describe this we need to move the description from the ‘number density of remnants’ to the actual number of discrete sources

$$\frac{\mathrm{d}^2 n}{\mathrm{d}z\,\mathrm{d}\mathcal{M}} = \frac{\mathrm{d}^3 N}{\mathrm{d}z\,\mathrm{d}\mathcal{M}\,\mathrm{d}\ln f_\mathrm{r}}\frac{\mathrm{d}\ln f_\mathrm{r}}{\mathrm{d}t_\mathrm{r}}\frac{\mathrm{d}t_\mathrm{r}}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}V_\mathrm{c}}$$



Here *df/dt* is given by GR and *dt/dz* and *dz/dV* are known relations from standard cosmology. Substituting we get

$$\boxed{h_c^2(f) = \int_0^\infty \mathrm{d}z \int_0^\infty \mathrm{d}\mathcal{M}\,\frac{\mathrm{d}^3 N}{\mathrm{d}z\,\mathrm{d}\mathcal{M}\,\mathrm{d}\ln f_\mathrm{r}}\,h^2(f_\mathrm{r})} \equiv \int_0^\infty \mathrm{d}z \int_0^\infty \mathrm{d}\mathcal{M}\,\frac{\mathrm{d}^3 N}{\mathrm{d}z\,\mathrm{d}\mathcal{M}\,\mathrm{d}t_\mathrm{r}}\frac{\mathrm{d}t_\mathrm{r}}{\mathrm{d}\ln f_\mathrm{r}}\,h^2(f_\mathrm{r})$$

Where *h* is the inclination polarization average strain $h(f_\mathrm{r}) = \sqrt{\frac{32}{5}}\frac{(G\mathcal{M})^{5/3}}{c^4 D}(\pi f_\mathrm{r})^{2/3}$

We can take a step further! Sources are discrete, so integrals here are actually sums. Moreover, if we observe for a time *T*, our frequency resolution bin is $\Delta f = 1/T$

The spectrum is therefore evaluated at discrete frequency bins $\Delta f_j = [j/T, (j+1)/T]$

And the characteristic strain is evaluated at discrete frequencies f_j as:

$$h_c^2(f_j) = \sum_z \sum_{\mathcal{M}} \frac{\Delta^3 N}{\Delta z \Delta \mathcal{M} \Delta f_j} f_j h^2(f_r) \Theta\left(\frac{f_r}{1+z}, \Delta f_j\right) \Delta z \Delta \mathcal{M}$$

A series of simplifications lead to:

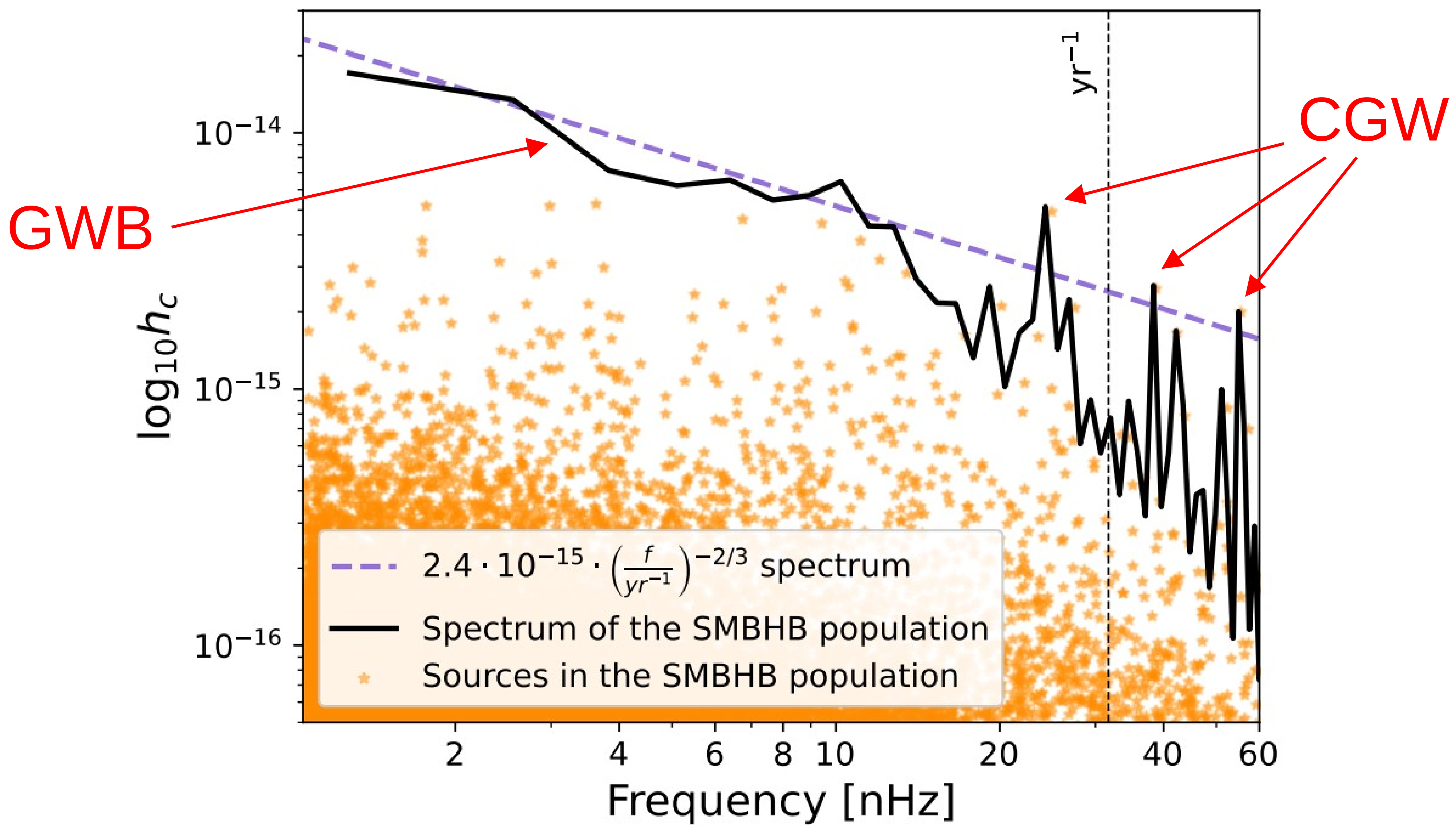
$$\boxed{h_c^2(f_j) = \sum_{f \in \Delta f_j} \frac{f_j h_i^2(f_r)}{\Delta f_j} = \sum_{f \in \Delta f_j} h_i^2(f_r) f_j T = \sum_{f \in \Delta f_j} h_i^2(f_r) N_{\rm cyc} = \sum_{f \in \Delta f_j} h_{c,i}^2(f_r)}$$

In practice h_c^2 is given by the sum in quadrature of the strain squares of each individual source multiplied by the number of cycles completed in the observation time

Since $dN/df \propto f^{-11/3}$ and $h_{c,i}^2 \propto f^{7/6}$ then 'on average' it is still true that $h_c \propto f^{-2/3}$ but:

-high *f*: many sources with small amplitued → stochastic GWB

-low *f*: few sources with large amplitude → individually resolvable CGWs ***(see Boris lecture)***



GWB
CGW
yr^{-1}
10^{-14}
10^{-15}
10^{-16}
$\log_{10} h_c$
2
4
6
8
10
20
40
60
Frequency [nHz]
$2.4 \cdot 10^{-15} \cdot \left(\frac{f}{yr^{-1}}\right)^{-2/3}$ spectrum
Spectrum of the SMBHB population
Sources in the SMBHB population

All beautiful, but the detectability?

Recall that PTAs are sensitive to the expectation value of the correlation

$$r_{ab} = \Gamma(\zeta_{ab}) \int_0^\infty df\, P_h(f)$$

We can therefore compute the magnitude of the RMS residual in each frequency bin as

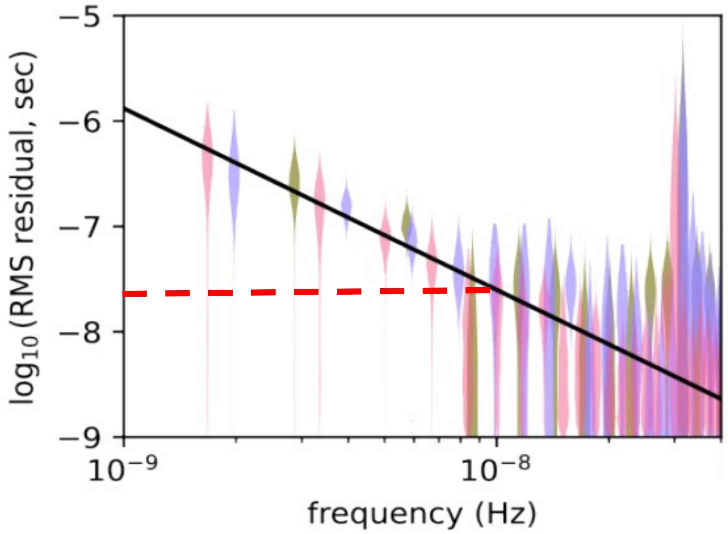
$$\mathrm{RMS}^2(f)_i = \int_{\Delta f_i} \mathrm{d}f\, P_h(f) \approx P_h(f_i)\Delta f_i = \frac{P_h(f_i)}{T}$$

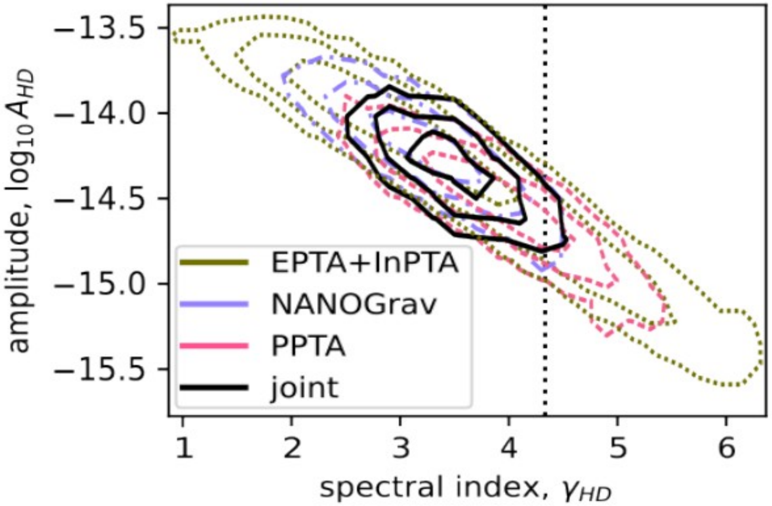
Where $P_h(f) = \frac{h_c^2(f)}{12\pi^2 f^3} = \frac{A_{\rm GWB}^2}{12\pi^2} \frac{f^\gamma}{(1\ \mathrm{year}^{-1})^{2\alpha}}$

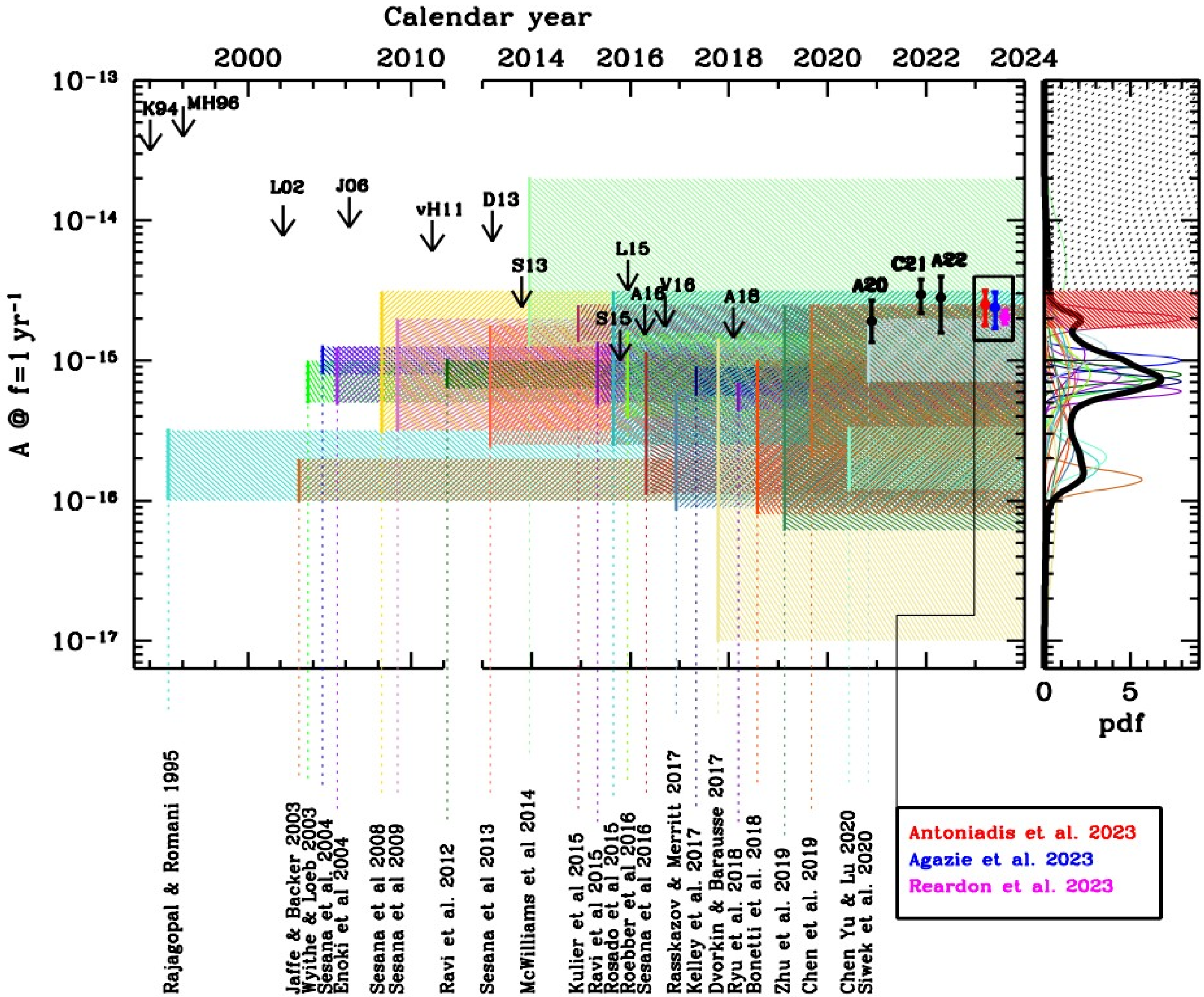
Here $\gamma = 3 - 2\alpha$ = 13/3 for α = -2/3

For $A_{\rm GWB}$ =10^{-15}, T=20 and f=10nHz, we get RMS~8ns

In the actual PTA data we get RMS~20ns, consistent with $A_{\rm GWB}$ =2.5×10^{-15}







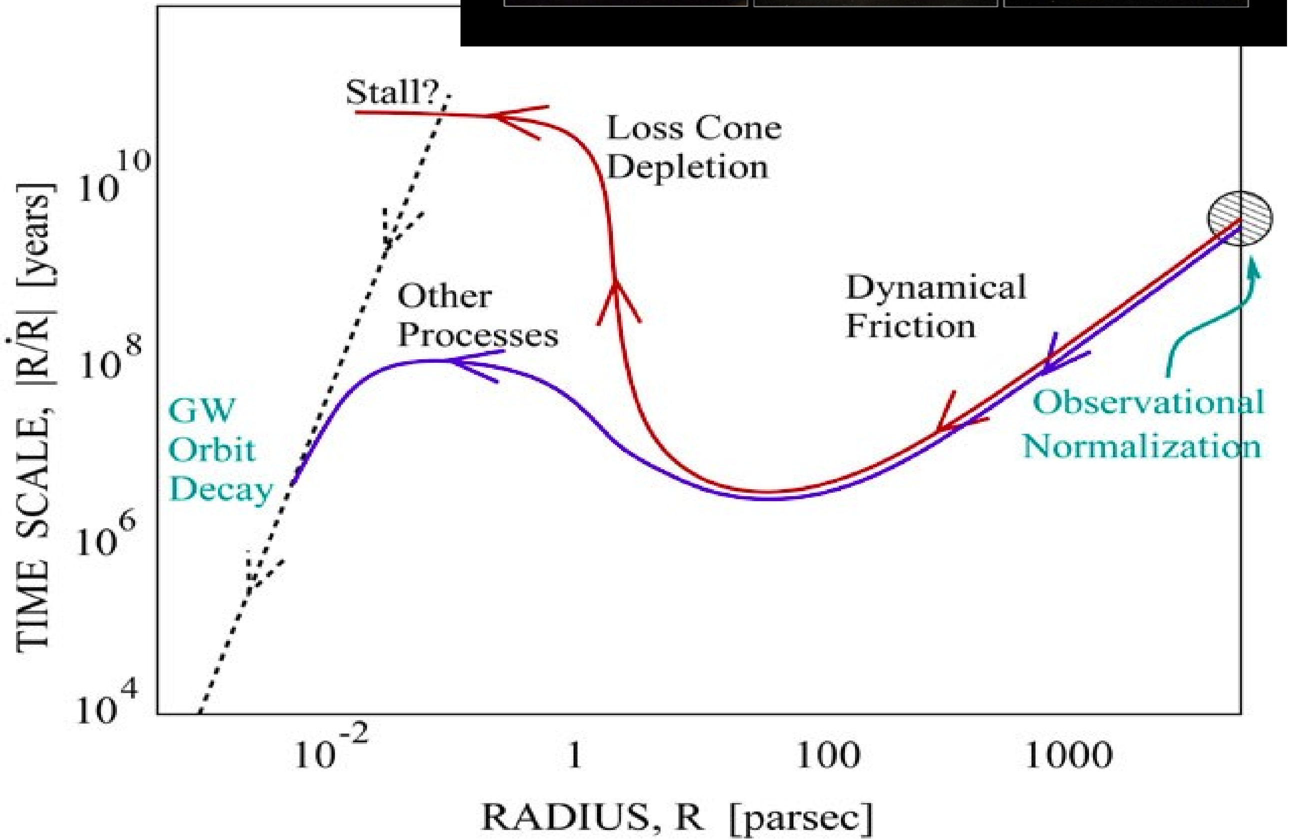
Calendar year
2000
2010
2014
2016
2018
2020
2022
2024
A @ f=1 yr^{-1}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
K94
MH96
L02
J06
vH11
D13
S13
L15
V16
A16
S15
A18
A20
C21
A22
0
5
pdf
Rajagopal & Romani 1995
Jaffe & Backer 2003
Wyithe & Loeb 2003
Sesana et al. 2004
Enoki et al 2004
Sesana et al 2008
Sesana et al 2009
Ravi et al. 2012
Sesana et al 2013
McWilliams et al 2014
Kulier et al 2015
Ravi et al 2015
Rosado et al 2015
Roebber et al 2016
Sesana et al 2016
Rasskazov & Merritt 2017
Kelley et al. 2017
Dvorkin & Barausse 2017
Ryu et al. 2018
Bonetti et al. 2018
Zhu et al. 2019
Chen et al. 2019
Chen Yu & Lu 2020
Siwek et al. 2020
Antoniadis et al. 2023
Agazie et al. 2023
Reardon et al. 2023

MBHB dynamics (BBR 1980)

$$f_{\mathrm{K},\star/\mathrm{GW}} \approx 5\times10^{-10}\left(\frac{M_1}{10^9\ \mathrm{M}_\odot}\right)^{-5/8}\left(\frac{\rho_{\inf}}{\rho_{\inf,\mathrm{SIS}}}\right)^{3/10} q^{-3/10}\mathcal{F}(e)^{-3/10}\ \mathrm{Hz},$$

$$f_{\mathrm{K,gas/GW}} \approx 5\times10^{-10}\left(\frac{\alpha}{0.1}\right)^{-6/49}\left(\frac{M_1}{10^9\ \mathrm{M}_\odot}\right)^{-37/49} q^{-69/98}\eta_{\mathrm{E}}^{-33/98}\mathcal{F}(e)^{-6/14}\ \mathrm{Hz}$$

GWs are efficient only at small separations, sub parsec binaries generally lose energy and angular momentum to the stellar and gaseous environment



GWB modified by:

-different dt/df due to hardening

-eccentricity of the binaries

GWB spectrum generalization

The GWB generalizes to:

$$h_c^2(f) = \int_0^\infty \mathrm{d}z \int_0^\infty \mathrm{d}M_1 \int_0^1 \mathrm{d}q \int_0^1 \mathrm{d}e_0 \frac{\mathrm{d}^5 N}{\mathrm{d}z\,\mathrm{d}M_1\,\mathrm{d}q\,\mathrm{d}t\,\mathrm{d}e_0} \left[\frac{\mathrm{d}t}{\mathrm{d}\ln f_{\mathrm{K,r}}} \frac{\mathrm{d}e_0}{\mathrm{d}e}\right] \times h^2(f_{\mathrm{K,r}}) \sum_{n=1}^{+\infty} \frac{g(n,e)}{(n/2)^2} \bigg|_{f_{\mathrm{K,r}}=f(1+z)/n}$$

Cosmic merger rate as a function of *M, q, e.*

Signal distribution across orbital period harmonics due to eccentricity

Residence time depending on the hardening process and eccentricity

Discretization leads to:

$$\boxed{h_c^2(f_j) = \sum_{i=1}^{N_B} \sum_{n=1}^{+\infty} h_i^2(f_{\mathrm{K,r}}) \frac{g(n,e_i)}{(n/2)^2} \frac{n f_{\mathrm{K,r}_i}}{\Delta f(1+z)} \Theta\left(\frac{n f_{\mathrm{K,r}_i}}{1+z}, \Delta f_j\right)}$$