

Noise Processes in Pulsar Timing Arrays

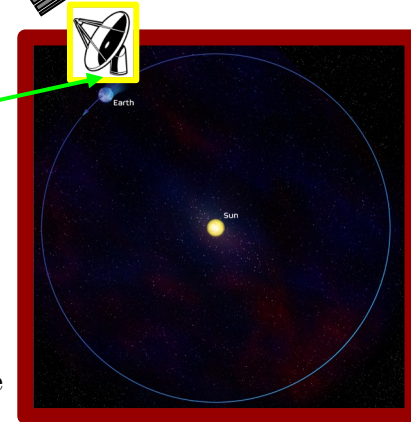
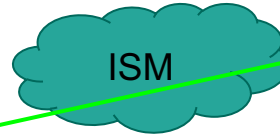
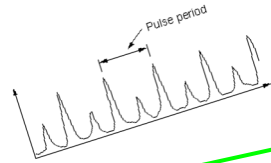
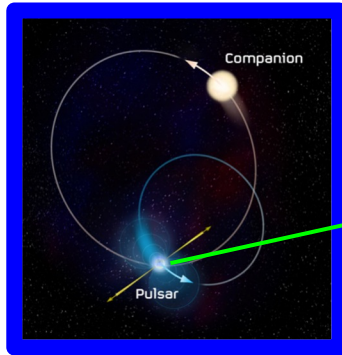
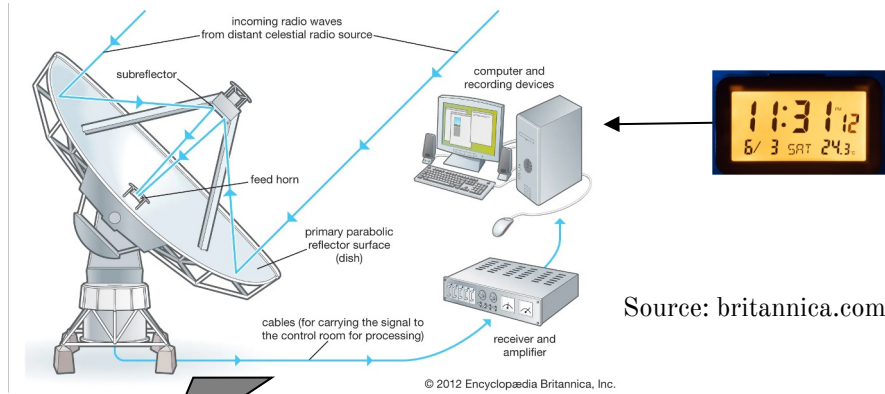


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Summary

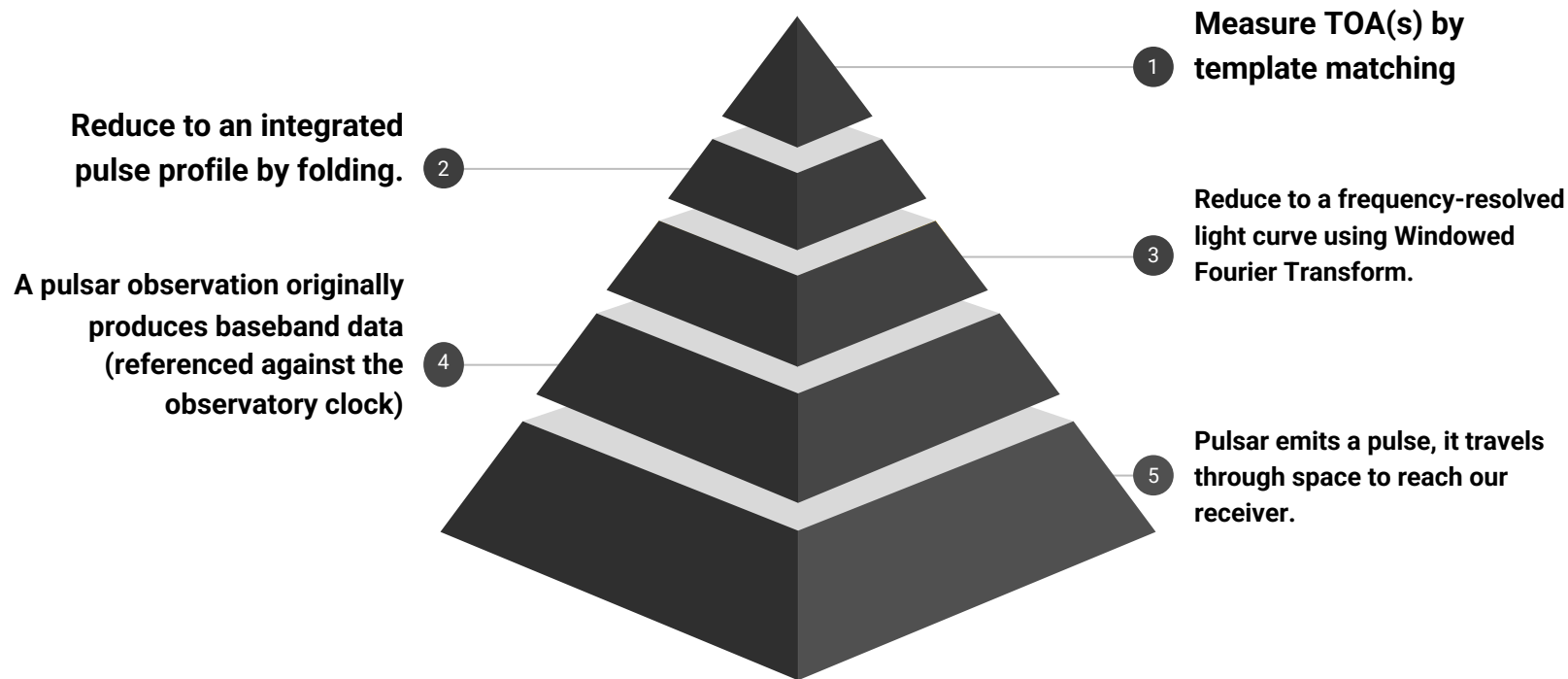


Quick Recap - How a TOA is made.



Credit: Paulo Freire

Quick Recap - How a TOA is made.



Introduction

- In PTA experiments, we measure pulsar TOAs to find gravitational wave signatures in them.
 - A radio PTA dataset contains TOAs of multiple pulsars measured at some observatory along with the measurement uncertainty.
- But TOAs are influenced by several astrophysical and instrumental effects other than gravitational waves.
- We must model all of these effects adequately to reliably detect gravitational waves.
 - Timing & Noise **model** — A mathematical description of all processes affecting measured TOAs.
 - Timing residuals — Measured TOA minus the TOA predicted by the model.

Noise is any effect influencing a TOA other than what we are currently trying to detect/study.

Scenario 1 — Detection of an Individual GW Source

- Pulsar rotation

- Spindown due to EM/GW emission
- Spin noise
- Glitches

- Pulsar emission

- Pulse jitter
- Mode changes
- Profile variability events

- Motion of the pulsar and the observatory

- Pulsar binary motion
- Proper motion
- Galactic motion
- Solar system motion
- Earth's rotation

- Astrophysical propagation effects

- Interstellar dispersion
- Interstellar scattering
- Solar wind dispersion

- Gravitational waves

- Gravitational wave background
- Individual GW source

- Instrumental effects

- Observing hardware and environment
 - Instrumental delays
 - Radiometer noise
 - Radio frequency interference
- Observing software
 - Data reduction, calibration, TOA measurement, ...
- Observatory clock

Scenario 2 — Test of General Relativity

- Pulsar rotation
 - Spindown due to EM/GW emission
 - Spin noise
 - Glitches
- Pulsar emission
 - Pulse jitter
 - Mode changes
 - Profile variability events
- Motion of the pulsar and the observatory
 - Pulsar binary motion
 - Proper motion
 - Galactic motion
 - Solar system motion
 - Earth's rotation
- Astrophysical propagation effects
 - Interstellar dispersion
 - Interstellar scattering
 - Solar wind dispersion
- Gravitational waves
 - Gravitational wave background
 - Individual GW sources
- Instrumental effects
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We can classify noise in pulsar timing arrays in different ways.

- Astrophysical vs Instrumental
- Pulsar-correlated vs Pulsar-uncorrelated
- Time-correlated vs Time-uncorrelated
- Achromatic vs Chromatic
 - Sometimes “achromatic” signals turn out to be actually chromatic! (Eg: Pulse jitter)
- Deterministic vs Stochastic
 - This distinction is often arbitrary! (Eg: IPS measurements of solar wind vs solar wind GP)

How do we deal with “noise” processes?

- In some cases we can simply remove the noise-affected parts of the data.
 - Eg: Radio frequency interference
- If we can measure the noise process (or an aspect of it) very precisely, we can directly use that information in pulsar timing.
 - Eg: Observatory clock corrections, Solar system ephemerides, Earth’s rotation history, Solar wind measurements using IPS
 - If there are errors in these measurements, they will show up as residual noise (Eg: clock errors, ephemeris errors)
- If we have no independent information about a noise process, we must estimate it from the data simultaneously with the signal.
 - Eg: Spin noise, interstellar dispersion
- If we have some information, but not precise enough, we can use them as informative prior distributions in Bayesian (timing and) noise analysis.
 - Eg: VLBI astrometric measurements, Optical/IR measurements of the pulsar binary

Timing Model vs Noise Model

- It is often convenient to split the Timing & Noise Model into a Timing Model and a Noise model.
- Timing Model consists of “big” effects
 - “Big” means that the contribution to the TOAs is comparable to or greater than the pulsar rotation period.
 - In practice, we compute the timing residuals using the Timing Model, while neglecting other effects.
 - Eg: Spindown, Solar system motion, Binary motion, large-scale trends in interstellar dispersion, etc.
- Noise Model consists of small deviations in the Timing Model + all other “small” effects not included in the timing model
 - “Small” means that the contribution of an effect on the TOAs is much smaller than the pulsar rotation period.
 - These effects are treated as **linear perturbations** on the TOAs, manifesting as non-zero timing residuals.
 - Eg: Radiometer noise, Pulse jitter, Dispersion variations, Spin noise, etc..
- **This separation is arbitrary and model-dependent.**

Components of the “Noise Model”



Gaussian Processes

A Gaussian process is a stochastic signal $\mathbf{y}$ given by

$$\mathbf{y} = \mathbf{y}_0 + \mathbf{T} \mathbf{b}$$

Where $\mathbf{b}$ has a *prior* probability distribution

$$\mathbf{b} \sim N[\mathbf{0}, \mathbf{B}]$$

For PTA applications, we choose the “basis matrix” $\mathbf{T}$ to be a rectangular matrix such that the dimensionality of $\mathbf{b}$ is much less than the dimensionality of $\mathbf{y}$. This makes the computations much more efficient.

Radiometer Noise

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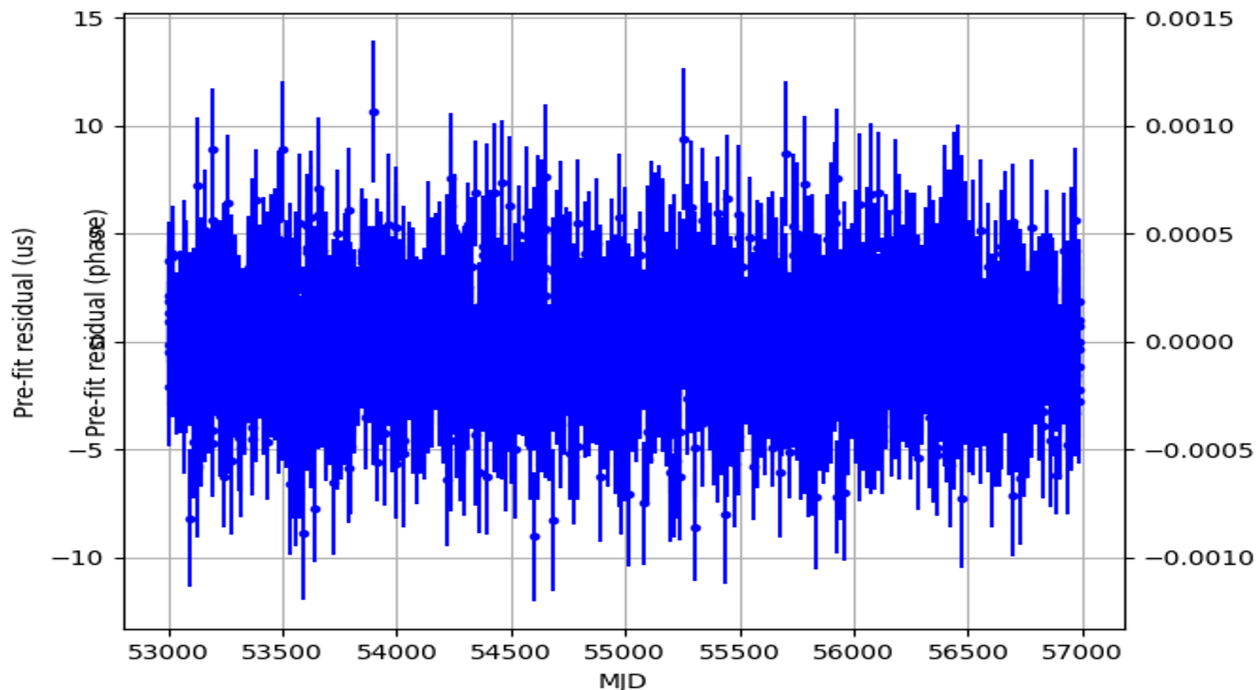
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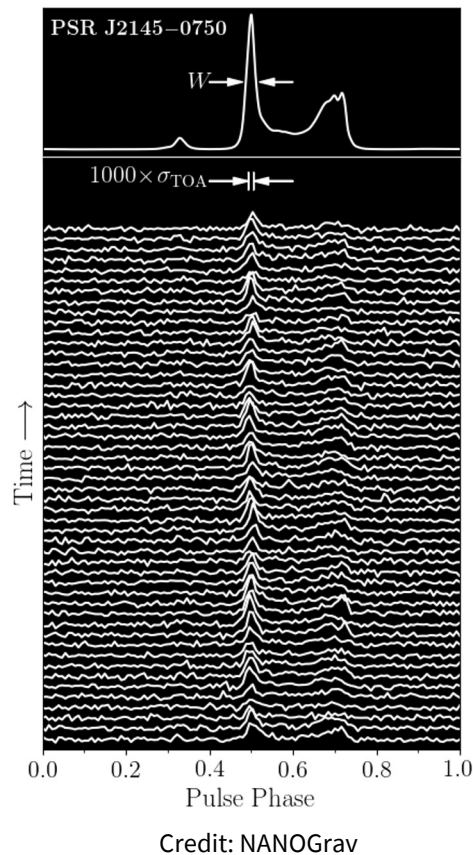
calibration,

Pulse Jitter

- The shape of individual pulses changes randomly
 - Although the profile shape is (mostly) constant if large number of pulses are folded.
 - In practice, we can only fold a finite number of pulses $\Rightarrow$ small pulse shape variations persist.
- Correlated across all TOAs measured from the same observation.
- Uncorrelated across pulsars
- Traditionally treated as achromatic, but recent studies show frequency-dependence.
- Depends on the observing system and its settings.
- Represented by the EQUAD and ECORR parameters.

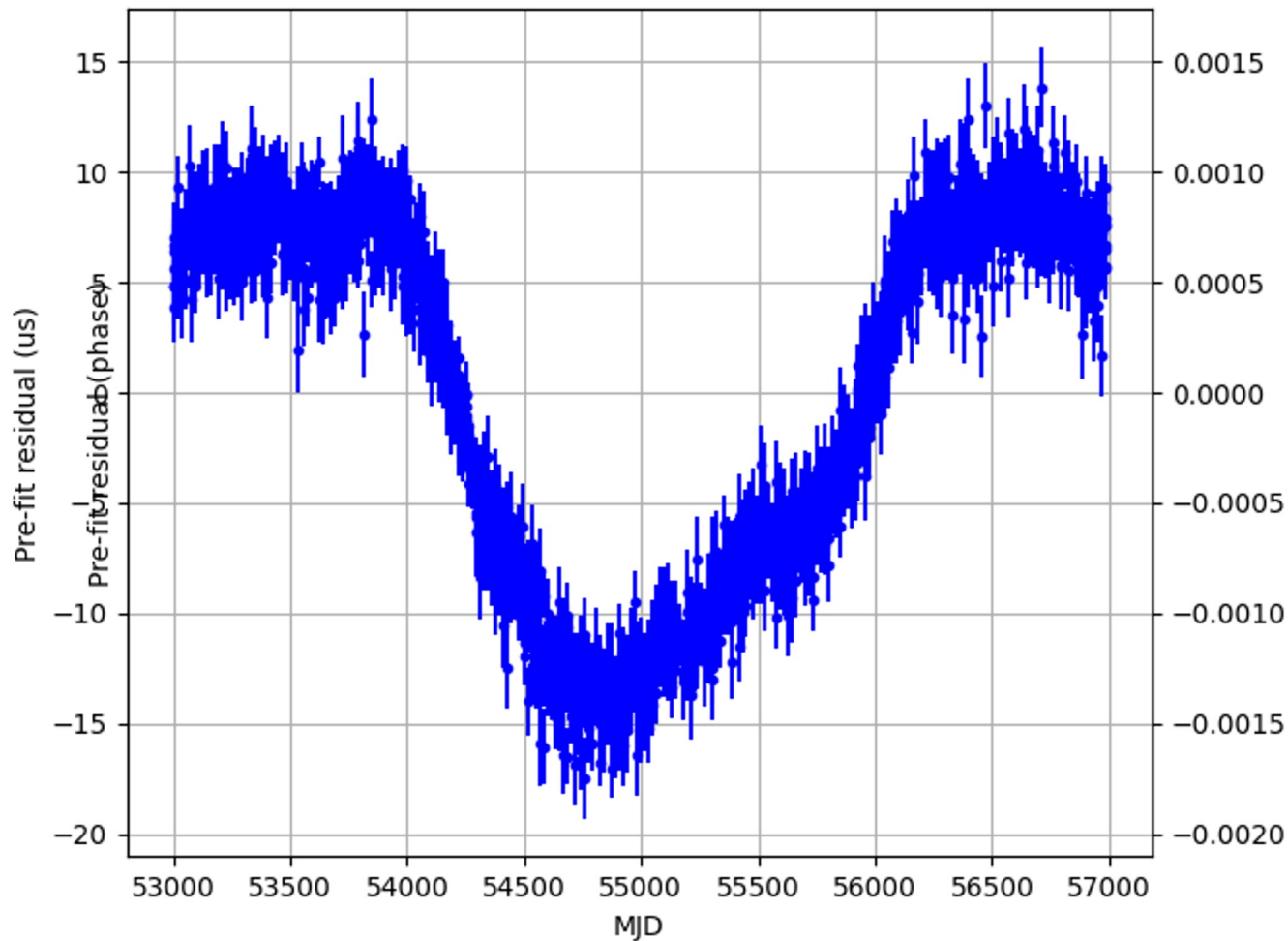
$$\text{Cov}[t_{ai}, t_{bj}] = \text{EFAC}^2 (\sigma_i^2 + \text{EQUAD}^2) \delta_{ab} \delta_{ij} + \text{ECORR}^2 \delta_{ab}$$

- Some profile-domain techniques for mitigating jitter noise have been proposed recently.



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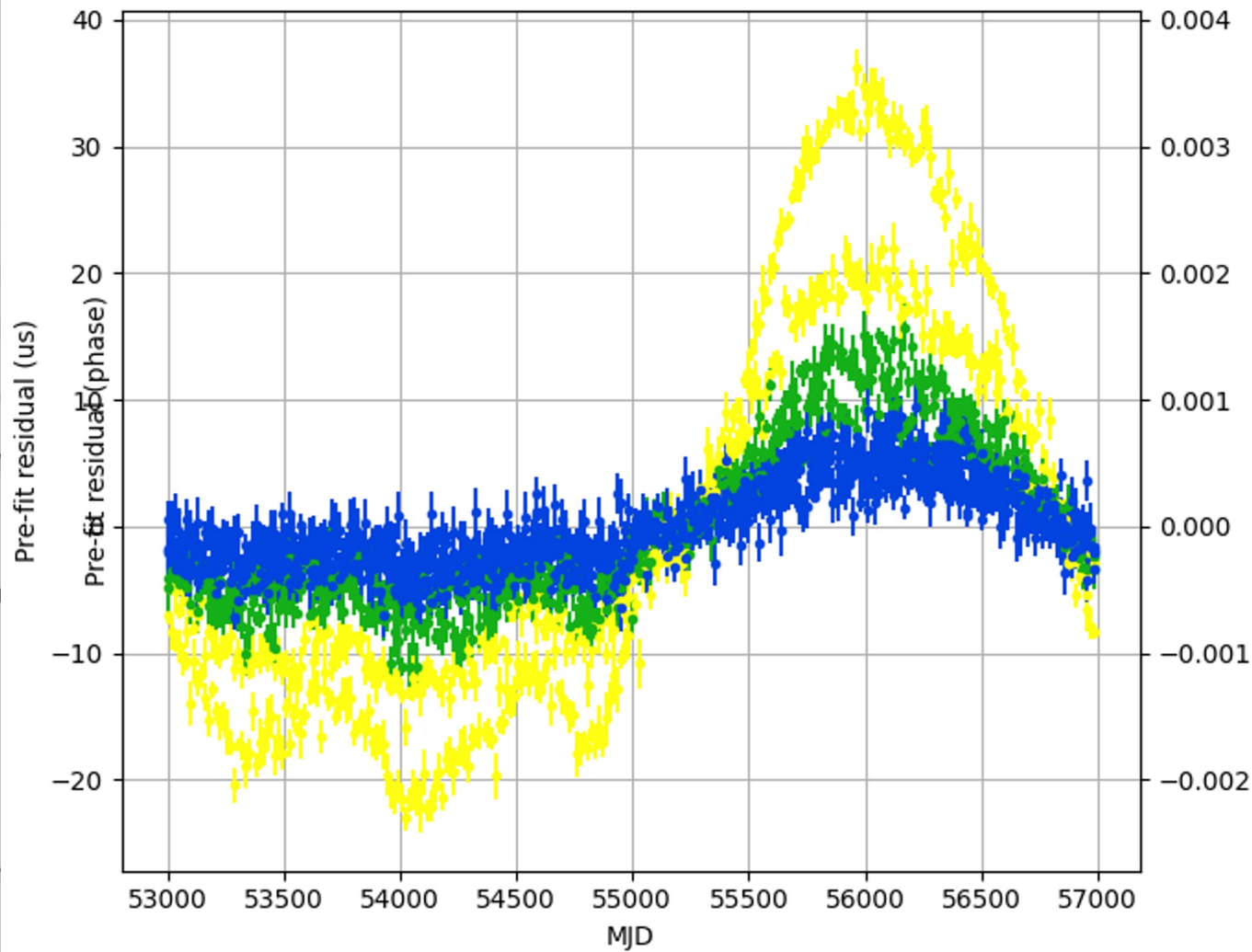
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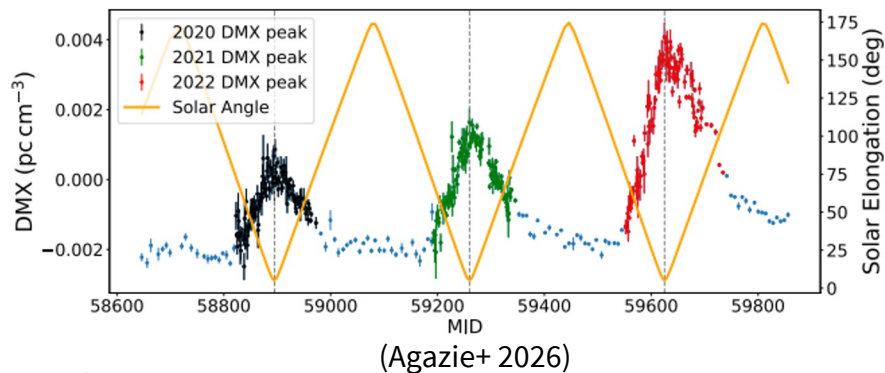
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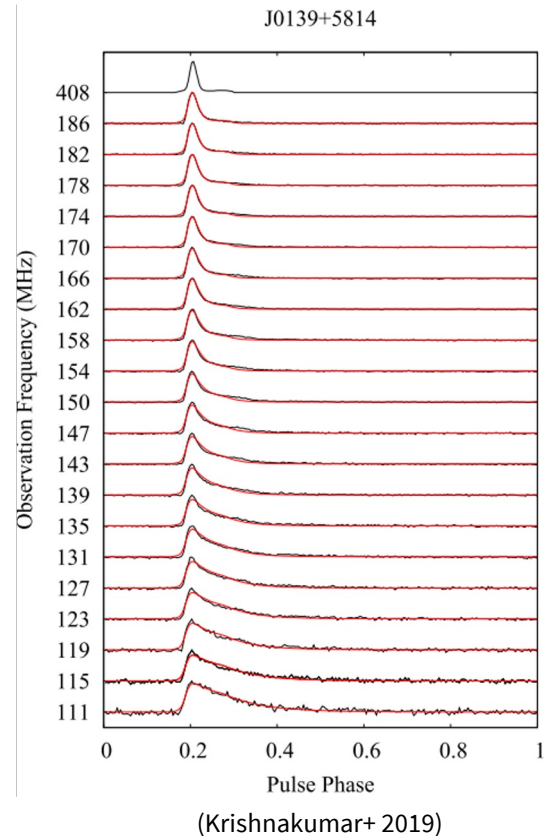
Solar Wind Noise

- Due to variable solar wind.
 - Charged particles in the solar wind introduces a frequency dependent refractive index.
- Time-correlated and pulsar-correlated.
 - Quasi-periodic with 1 year period.
 - Varies as a function of the solar elongation.
 - Pulsar correlations are not usually modeled (it's too hard?)
- Chromatic with a ν^{-2} dependence
- Doesn't depend on the observing system.
- Sometimes represented as a Fourier-domain Gaussian process with power-law spectrum.
 - Not a good model IMO $\neg_(\text{ツ})_/-$
- Can be mitigated by incorporating IPS measurements.



Scattering Noise

- Density fluctuations in the interstellar medium causes the pulsar radio signal to undergo multi-path propagation.
- This leads to systematic changes in the observed pulse shape.
- This violates the assumption that the profile shape is constant.
- Manifests as a chromatic, time-correlated noise in the TOAs.
 - Typically with an approximate $\nu^{-\alpha}$ dependence.
- Pulsar-uncorrelated, and doesn't depend on the observing system.
- No good TOA-domain models currently exist.
 - Most widely used model is a Fourier-domain Gaussian process with power-law spectrum and $\nu^{-\alpha}$ dependence
 - Promising directions — Cyclic spectroscopy (removes scattering in the baseband domain), CLEAN algorithm (removes scattering in the profile domain)



Binary Orbital Noise

- Some pulsars show complicated orbital variations.
 - Due to tidal interactions with the binary companion
 - Seen in redback and black widow systems.
 - Not common in PTA pulsars, but a few systems show this type of variations.
- Achromatic, Time-correlated, pulsar-uncorrelated.
- Doesn't depend on the observing system.
- Piecewise-constant and Fourier Gaussian process models have been used.
 - Not yet widely modeled in PTA analysis.

Solar System Ephemeris Noise

- Inaccuracies in the solar system ephemeris used to model solar system motion can manifest as a noise in PTA data.
- Was a bigger concern 5-6 years ago; ephemerides have since gotten much better.
- Time-correlated, pulsar-correlated (dipolar), and achromatic.
- Doesn't depend on the observing system.
- Can be modeled as a deterministic signal (BayesEphem) or as a Gaussian process (EphemGP)

Clock Noise

- Due to inaccuracies in the clock corrections used for converting the TOAs from the observatory time standard to an time standard defined at the solar system barycenter.
- Time-correlated, pulsar-correlated (monopolar), and achromatic.
- Depends on the telescope
- May be modeled as a deterministic signal or as a Gaussian process (depends on case-by-case)

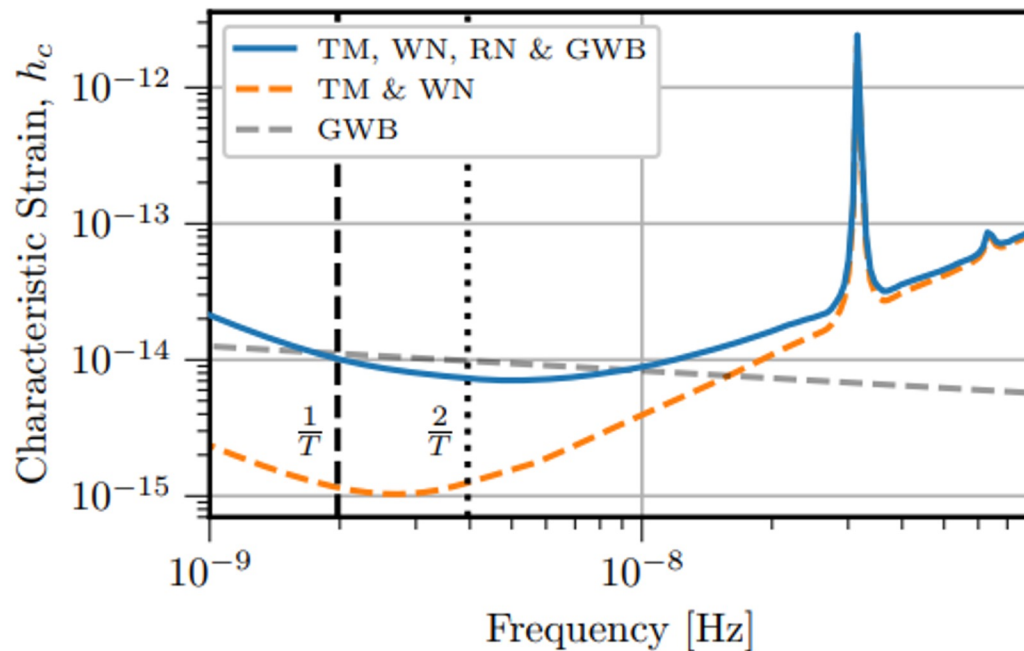
Gravitational Wave Background

- GWB acts as a noise if we are interested in detecting/studying another signal.
- Time-correlated, pulsar-correlated (quadrupolar), and achromatic.
- Doesn't depend on the observing system.
- Usually modeled as a Hellings-Downs correlated Fourier Gaussian process.
- You will hear more about this in other lectures.

Timing Model Deviations

- Small deviations in the timing model parameters obtained from the initial timing.
- Biggest effects are:
 - Unknown overall phase offset
 - Unknown rotational frequency and its derivatives
 - Unknown sky location and parallax
 - Unknown binary parameters if the orbital frequency is within the PTA band
- The component due to unknown sky location and parallax may be mitigated by incorporating VLBI measurements.
- Modeled as a Gaussian process with infinite prior variances.

PTA Sensitivity Curve



(Agazie+ 2023)

Bayesian (Timing &) Noise Analysis



Non-linear Single-Pulsar Timing and Noise Model

$$t_{\text{em}} = t_{\text{arr}} - \Delta_{\odot} - \Delta_{\text{SW}} - \Delta_{\text{ISM}} - \Delta_B - \dots$$

$$\phi = \phi_0 + \sum_{n=0}^{N_F} \frac{F_n}{(n+1)!} (t_{\text{em}} - t_0)^{n+1} + \dots$$

$$\mathbf{r} = \frac{\phi - N[\phi]}{\bar{F}}$$

$$\ln L = -\frac{1}{2} \mathbf{r}^T \mathfrak{C}^{-1} \mathbf{r} - \frac{1}{2} \ln \det[2\pi \mathfrak{C}]$$

$$\mathfrak{C} = \mathbf{N} + \mathbf{U}^T \Phi \mathbf{U}$$

$$P[\boldsymbol{\theta} | \{t_{\text{arr}}\}] \propto L[\{t_{\text{arr}}\} | \boldsymbol{\theta}] \times P[\boldsymbol{\theta}]$$

N contains the TOA (co)variances modified by EFACs, EQUADs, and ECORRs.

U contains the basis matrices for all Gaussian process components stacked column-wise

Φ contains the prior variances of each Gaussian process

θ contains all model parameters appearing in **r**, **N**, and **Φ**

Linearized (Timing and) Noise Model for a PTA

$$\ln \Lambda = -\frac{1}{2} (\mathbf{y} - \mathbf{s})^T \mathbf{C}^{-1} \mathbf{r} - \frac{1}{2} \ln \det[2\pi \mathbf{C}]$$

$$\mathbf{C} = \mathbf{N} + \mathbf{T}^T \mathbf{B} \mathbf{T}$$

- $\mathbf{y}$ contains the residuals for each pulsar
- $\mathbf{s}$ contains the total deterministic signal for each pulsar (not included in the timing model)
- $\mathbf{N}$ contains the modified TOA (co)variances for each pulsar (EFACs, EQUADs, and ECORRs)
- $\mathbf{T}$ is made by stacking the noise basis matrices and the timing model design matrices for each pulsar column-wise.
- $\mathbf{B}$ contains diagonal elements representing prior (co)variances for the Gaussian

Software Implementations

- Non-linear single-pulsar timing & noise analysis (SPNTA)
 - TEMPONEST
 - Vela.jl
- Linearized single-pulsar noise analysis (SPNA) / PTA analysis
 - ENTERPRISE
 - FortyTwo
 - Discovery
 - Prometheus

*Real data is
disgusting*

- George Hobbs

