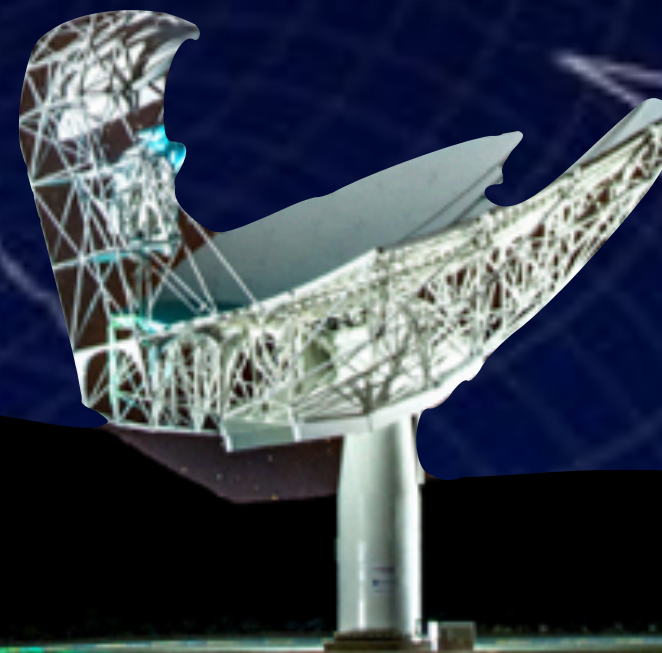


A (Brief) Introduction to Pulsars



High Energy Physics, Cosmology
& Astrophysics Theory



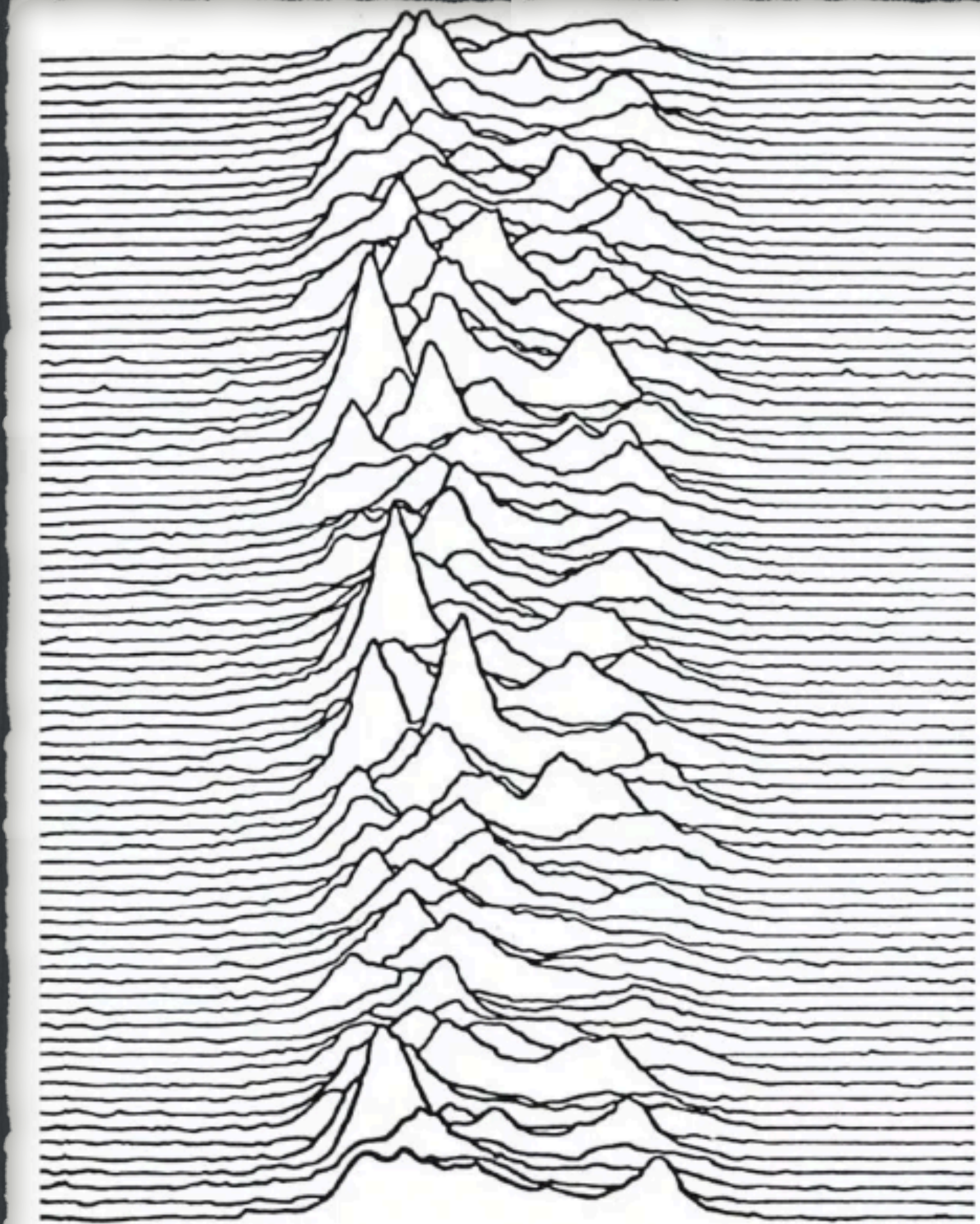
June 2026 | IPTA Student Week | NWU, Potchefstroom



Part 1: Pulsar Properties

Topics we will cover

- The discovery of CP1919
- The minimum density spinning object
- How pulsars are born
- The interior of a neutron star
- Neutron star radius and masses
- Standard pulsar model
- NS magnetic field
- Magnetic dipole radiation
- NS Rotational energy and loss of rotational energy
- Characteristic age
- The Pulsar radio population (P-Pdot diagram)





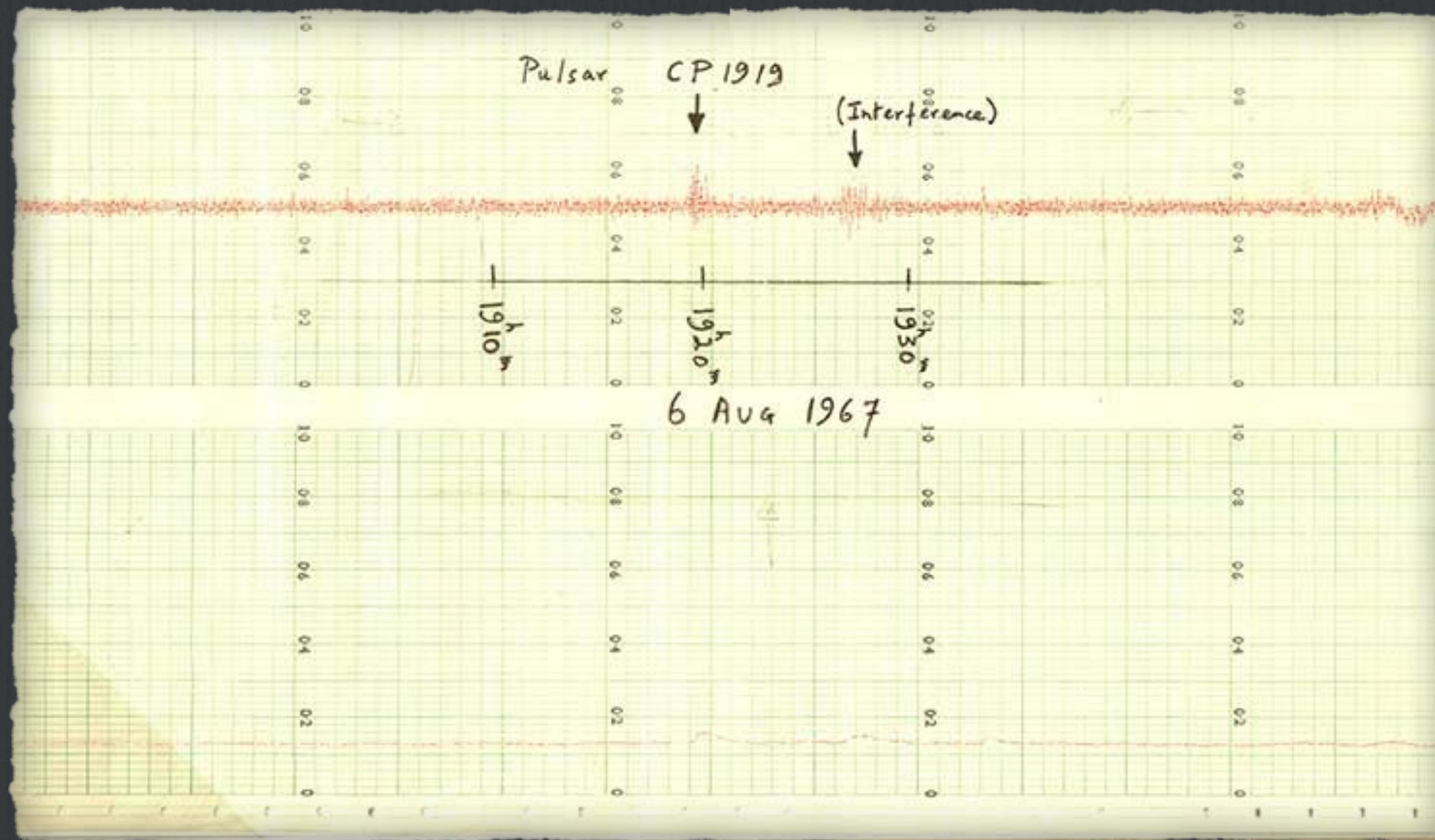
The Discovery of CP1919

- One of the great serendipitous discoveries in astronomy
- In the 1960s Jocelyn Bell-Burnell (then a PhD student at Cambridge Uni) was helping to build a new radio telescope in Cambridge
- The real aim of the experiment was to find extra galactic compact radio sources that scintillate in the interplanetary plasma
- In 1967 she noticed some 'scruffy' pulses in the chart recordings.



Image credit: University of Cambridge

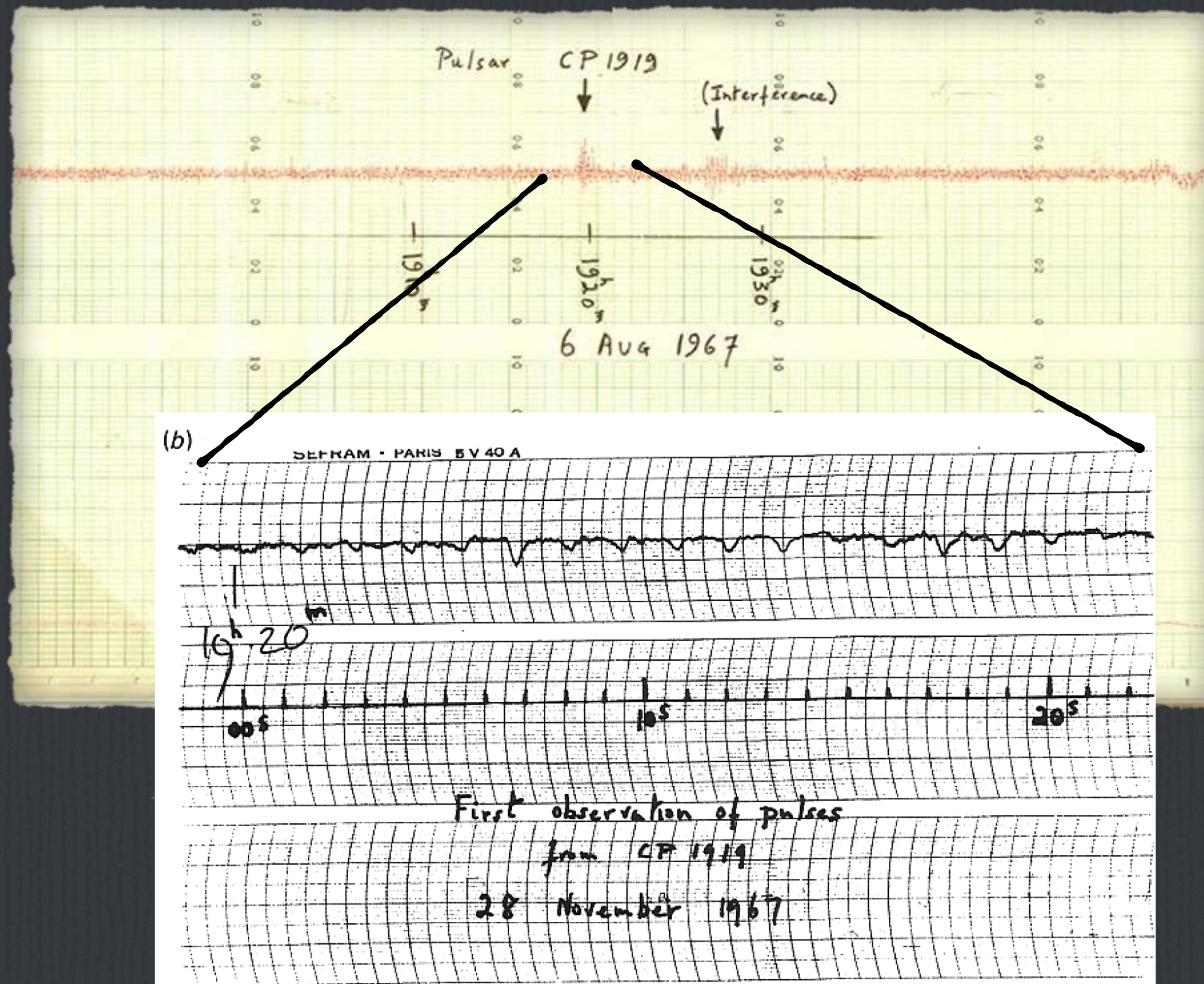
The Discovery of CP1919



- One of the great serendipitous discoveries in astronomy
- In the 1960s Jocelyn Bell-Burnell (then a PhD student at Cambridge Uni) was helping to build a new radio telescope in Cambridge
- The real aim of the experiment was to find extra galactic compact radio sources that scintillate in the interplanetary plasma
- In 1967 she noticed some 'scruffy' pulses in the chart recordings.

Image credit: University of Cambridge

The Discovery of CP1919



- The data were recorded using pen charts
- By running the chart recorder at a higher speed Jocelyn Bell-Burnell was able to separate the 'scruff' into distinguishable pulses
- The pulses appeared to have a period of 1.33 s
- They seemed oddly 'man made' — but they appeared to keep sidereal time, so it could not be terrestrial
- Perhaps it was signatures from LGM: 'little green men'

CP1919 - Could it be a spinning star?

Density of sphere — (1)

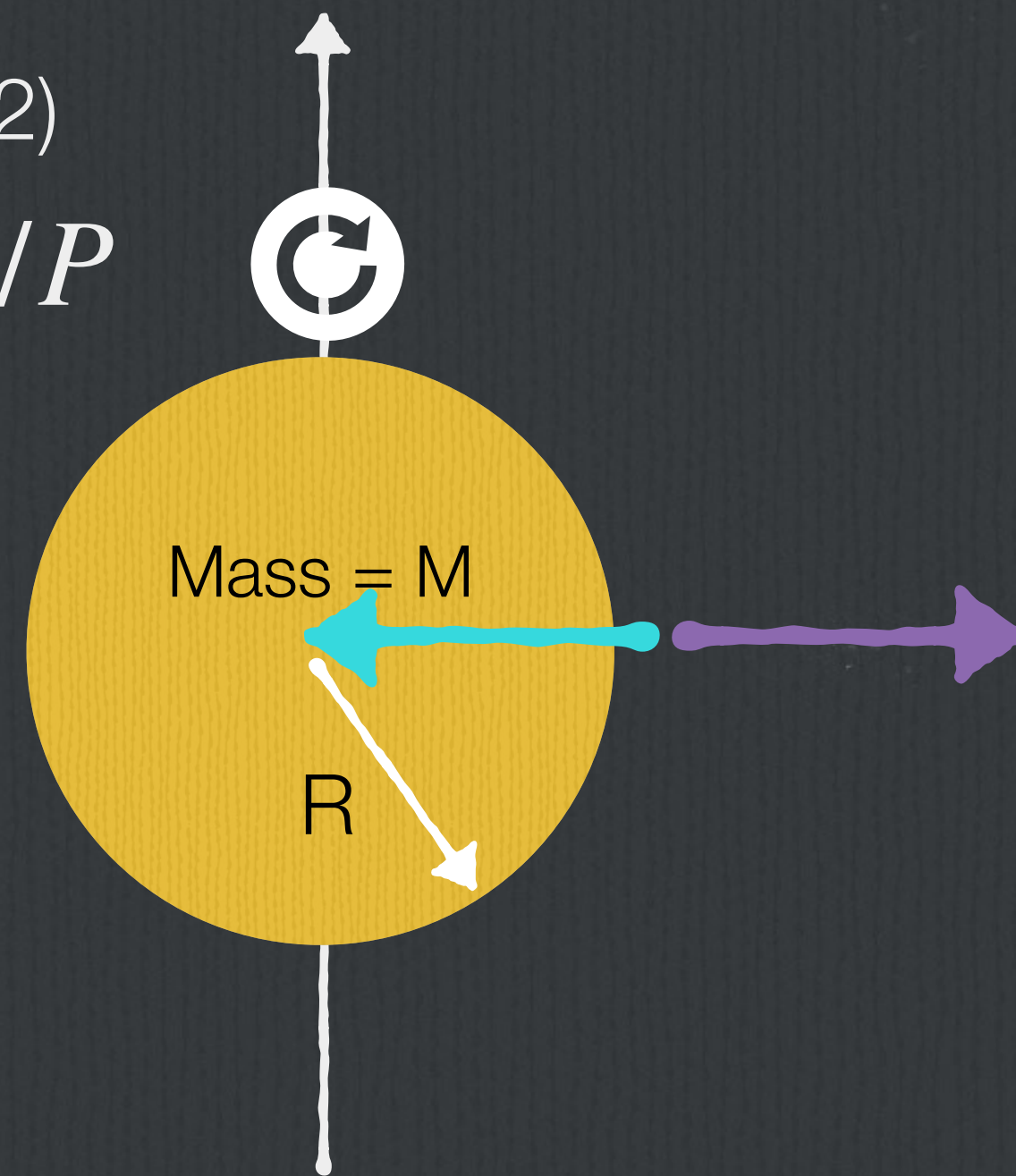
$$\rho = \frac{M}{V_{\text{sphere}}} = M \left(\frac{4}{3} \pi R^3 \right)^{-1}$$

- Pulses with period $P = 1.33\text{s}$
- Can this be a spinning star?
- The fastest a star can spin is set by the limit that

centrifugal acceleration (at the equator) < gravitational acceleration

Angular velocity — (2)

$$\Omega = 2\pi/P$$



$$\Omega^2 R < \frac{GM}{R^2}$$

CP1919 minimum stellar density

Density of sphere — (1)

$$\rho = \frac{M}{V_{\text{sphere}}} = M \left(\frac{4}{3} \pi R^3 \right)^{-1}$$

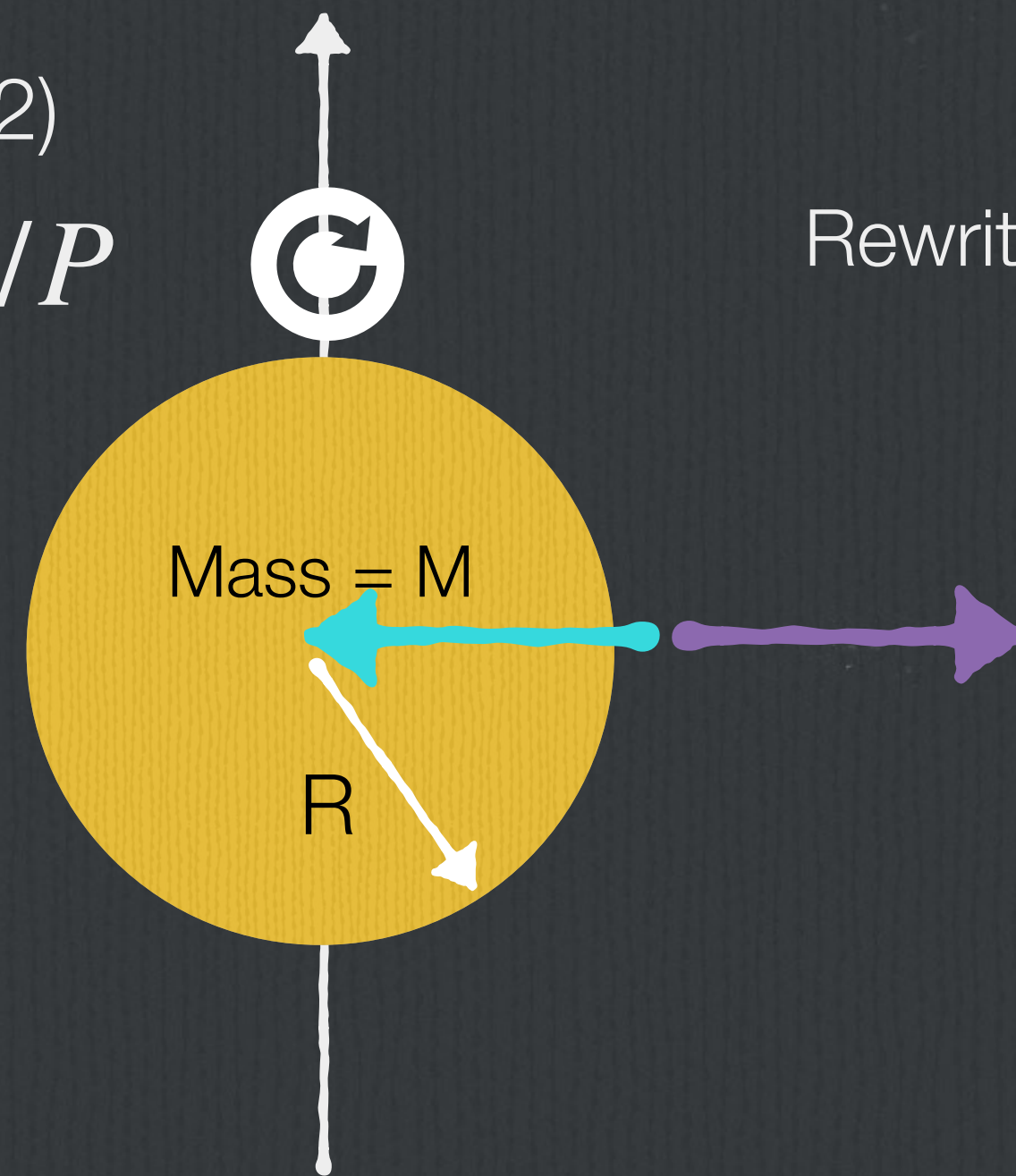
- Pulses with period $P = 1.33\text{s}$
- Can this be a spinning star?
- The fastest a star can spin is set by the limit that

centrifugal acceleration (at the equator) < gravitational acceleration

$$\Omega^2 R < \frac{GM}{R^2}$$

Angular velocity — (2)

$$\Omega = 2\pi/P$$



Rewriting i.t.o stellar density using (1) and (2):

$$\rho > \frac{3\pi}{P^2 G}$$

CP1919 minimum stellar density

Density of sphere — (1)

$$\rho = \frac{M}{V_{\text{sphere}}} = M \left(\frac{4}{3} \pi R^3 \right)^{-1}$$

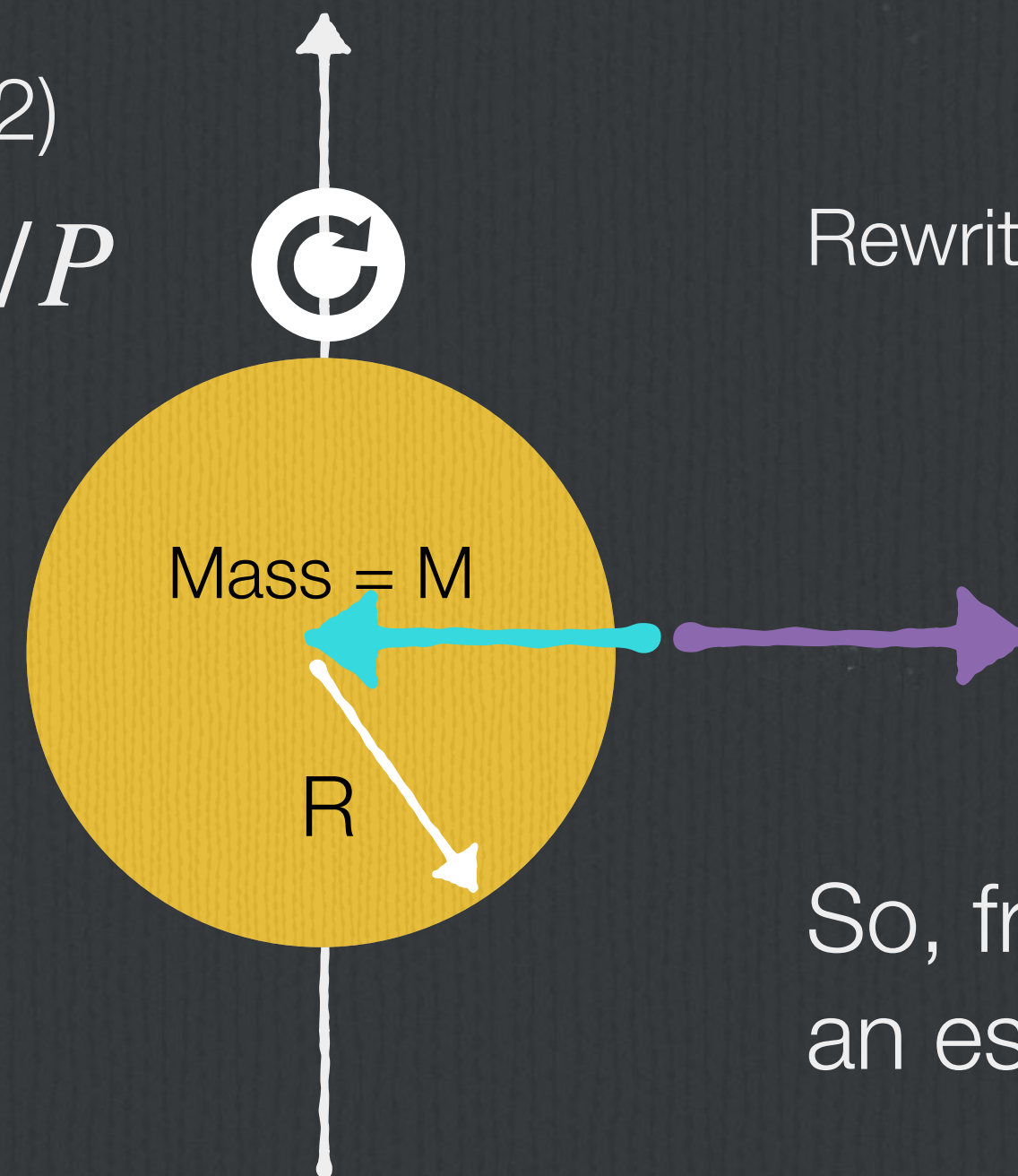
- Pulses with period $P = 1.33\text{s}$
- Can this be a spinning star?
- The fastest a star can spin is set by the limit that

centrifugal acceleration (at the equator) < gravitational acceleration

$$\Omega^2 R < \frac{GM}{R^2}$$

Angular velocity — (2)

$$\Omega = 2\pi/P$$



Rewriting i.t.o stellar density using (1) and (2):

$$\rho > \frac{3\pi}{P^2 G}$$

So, from only measuring the pulse period, we already have an estimate of the minimum density of the 'spinning object'

CP1919 minimum stellar density

- Pulses with period $P = 1.33\text{s}$
- The minimum density estimate of the spinning object:

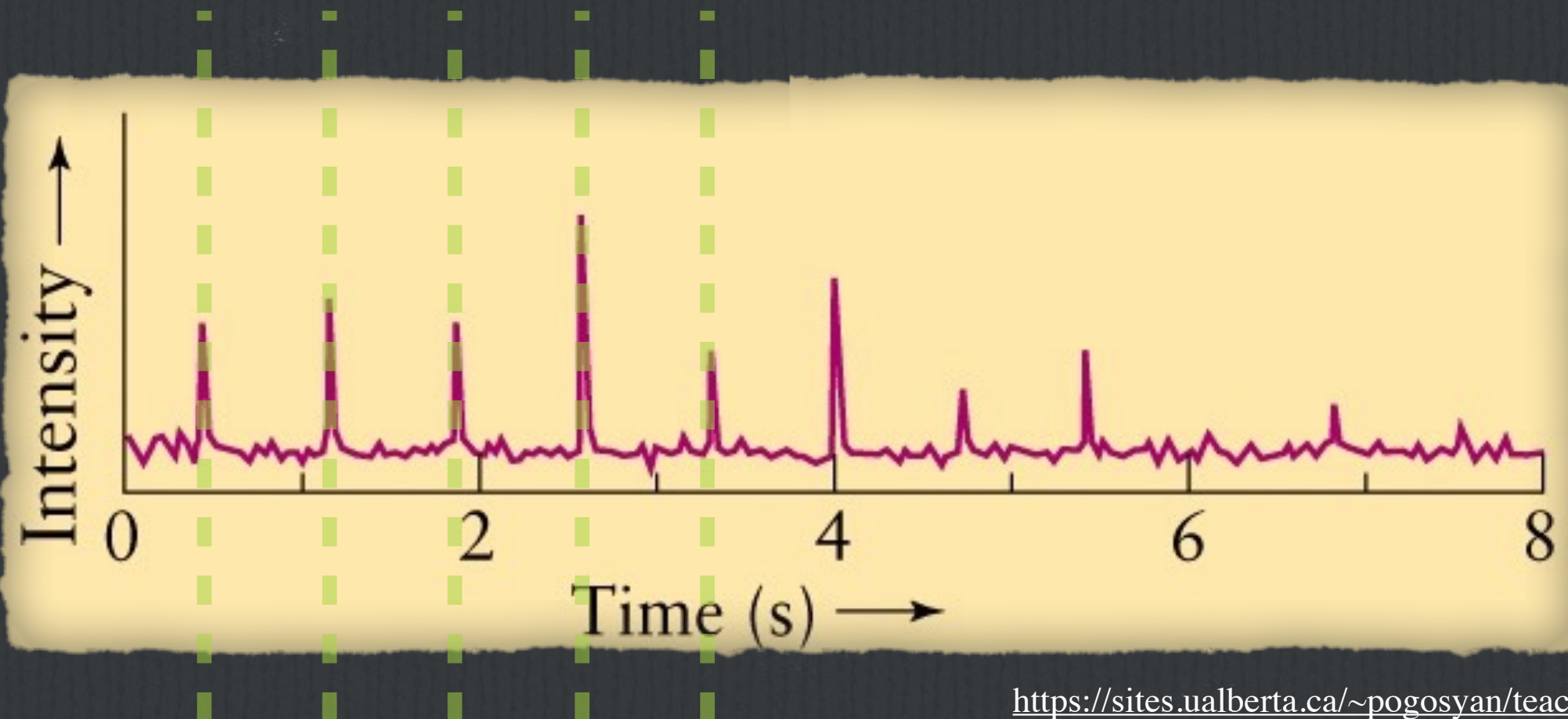
$$\rho > \frac{3\pi}{P^2 G}$$

$$G \sim 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$\rho > \frac{3\pi}{(1.33\text{s})^2 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}}$$

$$\rho \gtrsim 8 \times 10^{10} \text{ kg m}^{-3}$$

$P = 1.33 \text{ s}$



CP1919 minimum stellar density

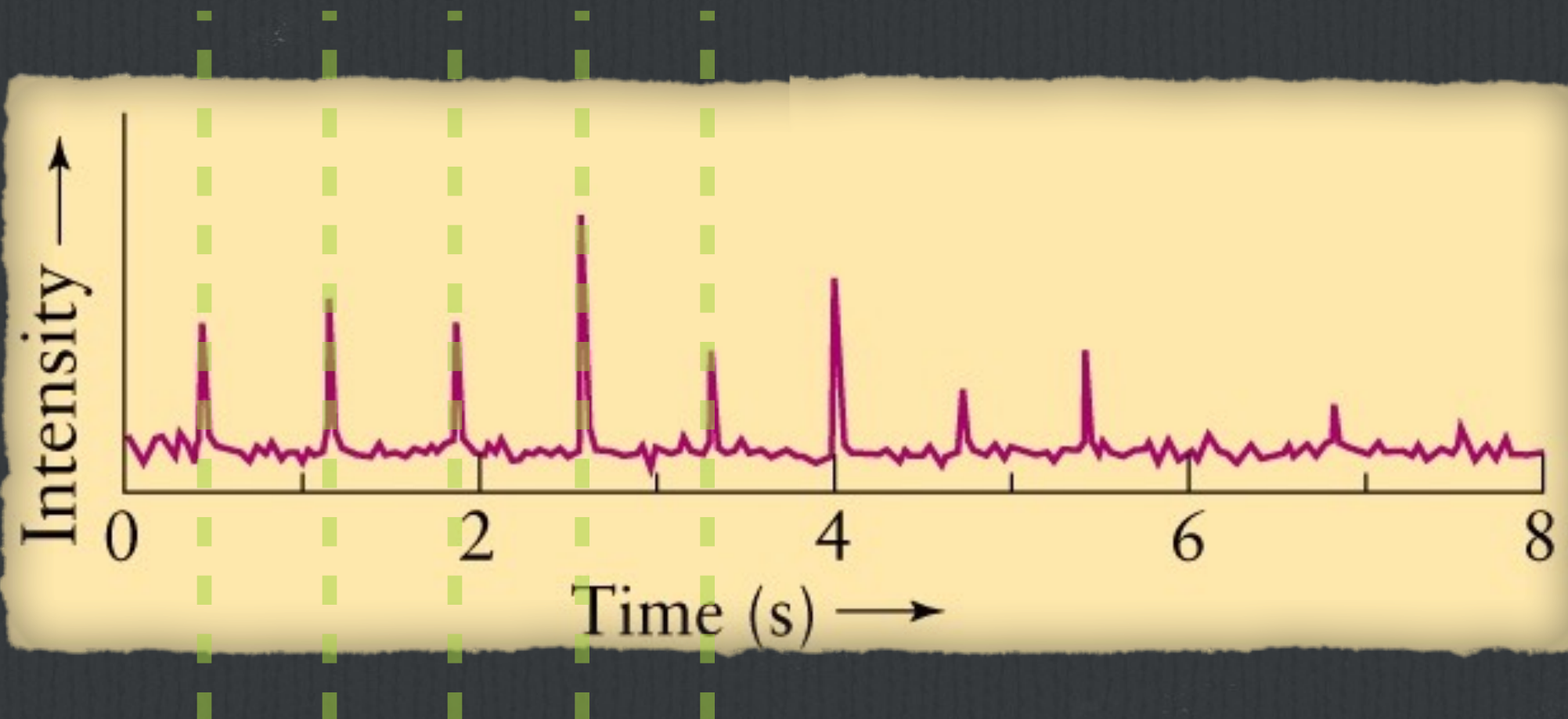
- Pulses with period $P = 1.33\text{s}$
- The minimum density estimate of the spinning object:

$$\rho > \frac{3\pi}{P^2 G}$$

$$G \sim 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

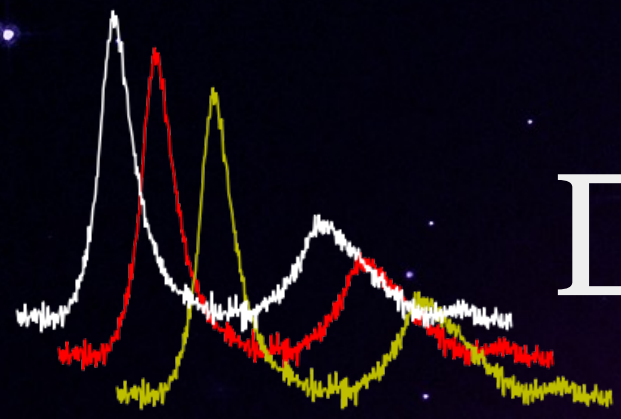
$$\rho \gtrsim 8 \times 10^{10} \text{ kg m}^{-3}$$

$$P = 1.33 \text{ s}$$



White dwarf density lies between 10^7 and $10^{11} \text{ kg m}^{-3}$

This means CP1919 was just within the white dwarf limit:
could it be a white dwarf?



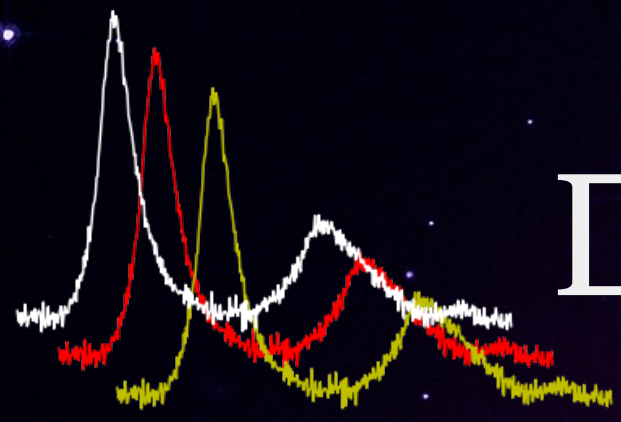
Discovery of the Crab pulsar

- Several pulsars were discovered shortly after CP1919
- Amongst them the Crab pulsar (PSR B0531+21) in 1968, associated with the Supernova remnant SN 1054
- With a spin period $P = 0.033\text{s}$ (33ms), the Crab pulsar has a minimum density of

$$\rho > \frac{3\pi}{(0.033\text{s})^2 6.67 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}}$$

$$\rho \gtrsim 1.3 \times 10^{14} \text{kg m}^{-3}$$

1000 x more dense



Discovery of the Crab pulsar

- Several pulsars were discovered shortly after CP1919
- Amongst them the Crab pulsar (PSR B0531+21) in 1968, associated with the Supernova remnant SN 1054
- With a spin period $P = 0.033\text{s}$ (33ms), the Crab pulsar has a minimum density of

$$\rho > \frac{3\pi}{(0.033\text{s})^2 6.67 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}}$$

$$\rho \gtrsim 1.3 \times 10^{14} \text{kg m}^{-3}$$

1000 x more dense

Too dense for a white dwarf, but ...fitted the theoretical description proposed by Baade and Zwicky 1934 of a 'neutron star' born through a supernova

Baade and Zwicky 1934

Baade and Zwicky published two papers in 1934 - back to back in PNAS (Proceedings of National Academy of Sciences):

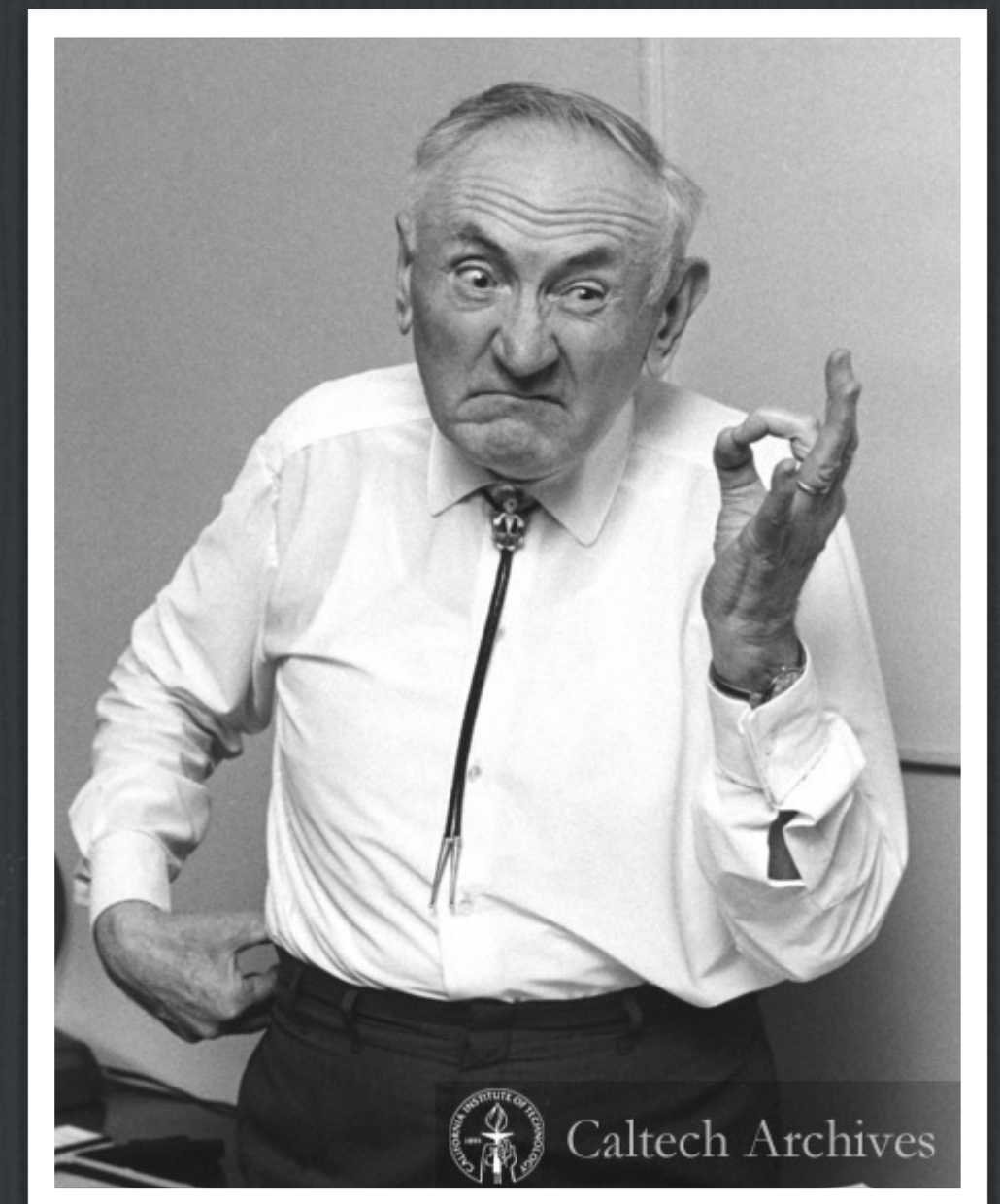
“On Super-novae” and “Cosmic Rays from Super-novae”,
in the “additional remarks” section propose

"With all reserve we advance the view that a super-nova represents the transition of an ordinary star into a **neutron star**, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density"

Walter Baade



Fritz Zwicky



How Pulsars are born

Depending on a star's mass, it will end its life differently — the most massive ones ($> 60 M_{\text{sun}}$) will collapse to black holes,

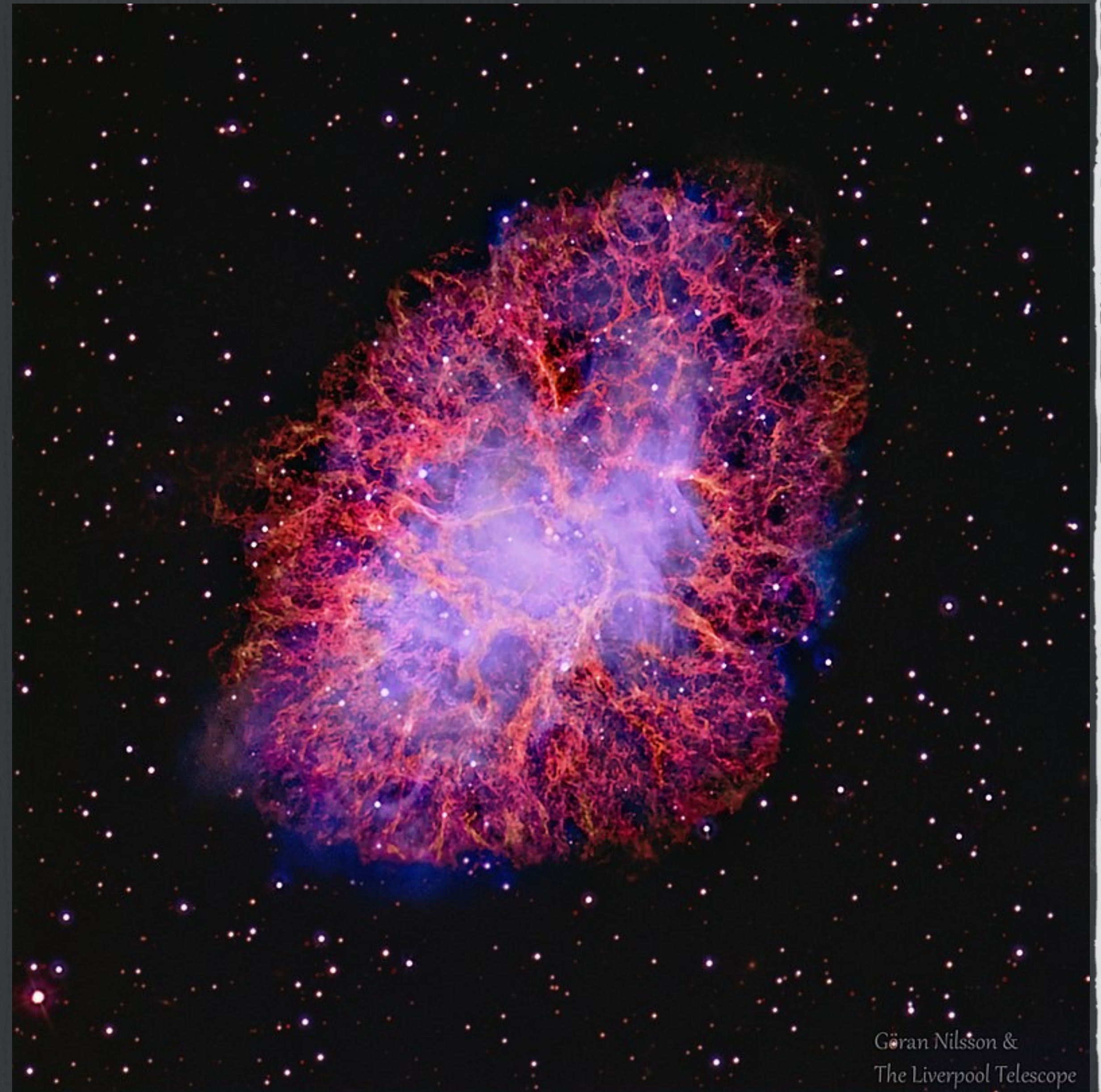
While the intermediate ones ($10 - 40 M_{\text{sun}}$) will end their lives collapsing to a dense core, stopped only by neutron degeneracy pressure.

Forming dense corpses (neutron stars) with radius $\sim 10\text{km}$ and masses $\sim 1.4 M_{\text{sun}}$

In the rebound shock the outer gas shells are flung out in a spectacular supernova explosion

EXTREME dead stars

- spin around their axis at up to 700 times a second
- $\sim 10\text{x}$ nuclear densities
- magnetic fields $>$ a trillion times that of Earth



Göran Nilsson &
The Liverpool Telescope

How Pulsars are born

Depending on a star's mass, it will end its life differently — the most massive ones ($> 60 M_{\text{sun}}$) will collapse to black holes,

While the intermediate ones ($10 - 40 M_{\text{sun}}$) will end their lives collapsing to a dense core, stopped only by neutron degeneracy pressure.

Forming dense corpses (neutron stars) with radius $\sim 10\text{km}$ and masses $\sim 1.4 M_{\text{sun}}$

In the rebound shock the outer gas shells are flung out in a spectacular supernova explosion

EXTREME dead stars

- spin around their axis at up to 700 times a second
- $\sim 10\text{x}$ nuclear densities
- magnetic fields $>$ a trillion times that of Earth



Object 4 times the height of Mt Kili,
whizzing like a kitchen blender!

Neutron star radius limit

Recall that the largest mass at which a white dwarf can be stable is the Chandrasekhar limit.

At this limit the star is supported by outward electron degeneracy pressure

$$M_{\text{Ch}} \sim \left(\frac{\hbar c}{G} \right)^{3/2} \frac{1}{m_{\text{p}}^2} \approx 1.4 M_{\odot}$$

Maximum Radius:

- From the simplified definition of density:

$$\rho = \frac{M}{V_{\text{sphere}}} = M \left(\frac{4}{3} \pi R^3 \right)^{-1} \rightarrow R < \left(\frac{3 M}{4 \pi \rho} \right)^{1/3}$$



Subrahmanyan Chandrasekhar (born 1910)

Source: Chandra X-ray Observatory

Neutron star radius limit

Recall that the largest mass at which a white dwarf can be stable is the Chandrasekhar limit.

At this limit the star is supported by outward electron degeneracy pressure

$$M_{\text{Ch}} \sim \left(\frac{\hbar c}{G} \right)^{3/2} \frac{1}{m_{\text{p}}^2} \approx 1.4 M_{\odot}$$

Maximum Radius:

- From the simplified definition of density:

$$\rho = \frac{M}{V_{\text{sphere}}} = M \left(\frac{4}{3} \pi R^3 \right)^{-1} \rightarrow R < \left(\frac{3 M}{4 \pi \rho} \right)^{1/3} \lesssim 20 \text{ km}$$



Subrahmanyan Chandrasekhar (born 1910)
Source: Chandra X-ray Observatory

$$\sim 7.2 \times 10^{16} \text{ kg m}^{-3}$$

Highest estimated density from fastest spinning pulsar

Neutron star radius limit

Recall that the largest mass at which a white dwarf can be stable is the Chandrasekhar limit.

At this limit the star is supported by outward electron degeneracy pressure

$$M_{\text{Ch}} \sim \left(\frac{\hbar c}{G} \right)^{3/2} \frac{1}{m_{\text{p}}^2} \approx 1.4 M_{\odot}$$

Maximum Radius:

- From the simplified definition of density:

$$\rho = \frac{M}{V_{\text{sphere}}} = M \left(\frac{4}{3} \pi R^3 \right)^{-1} \rightarrow R < \left(\frac{3 M}{4 \pi \rho} \right)^{1/3} \lesssim 20 \text{ km}$$

Minimum Radius:

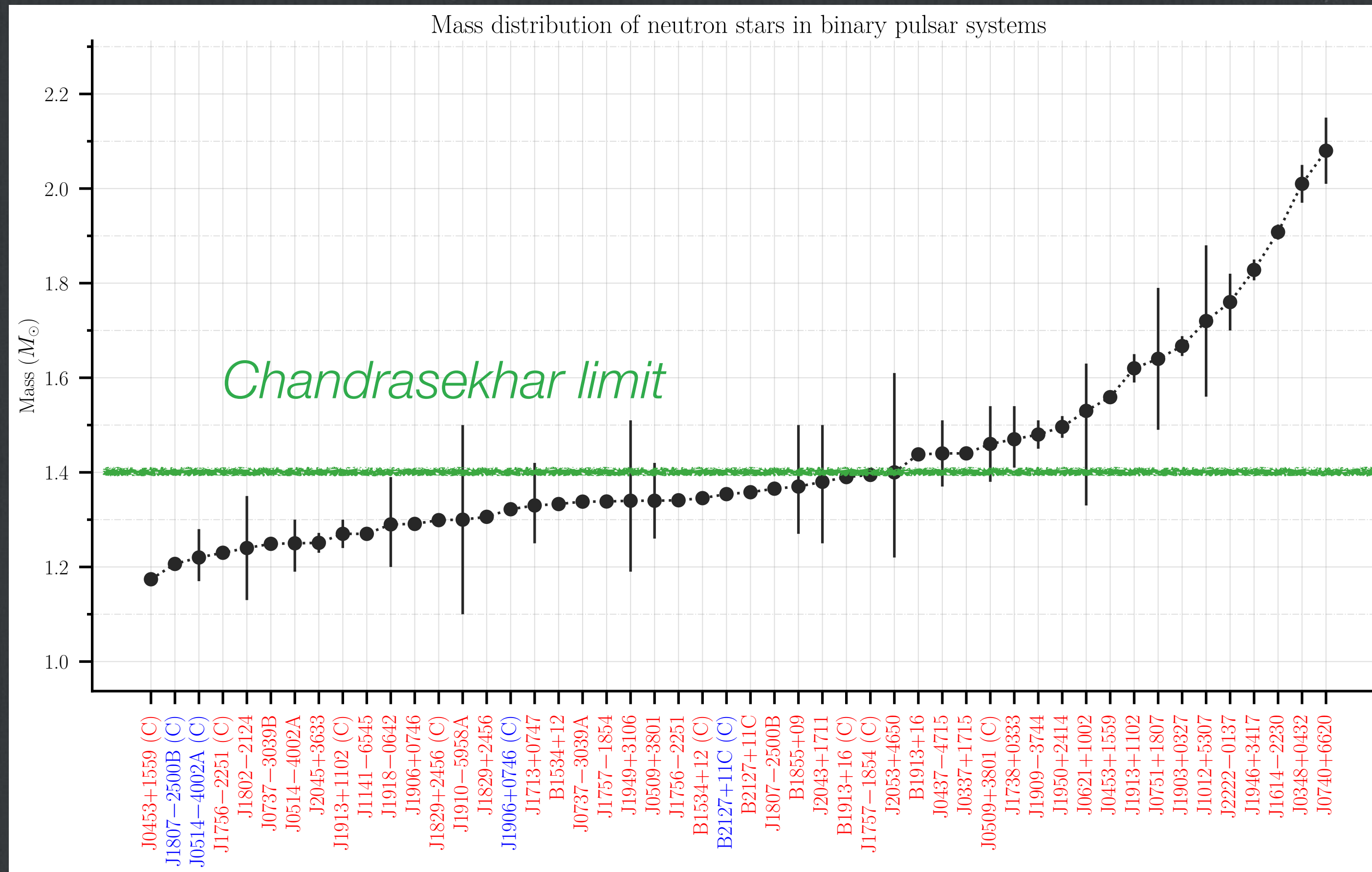
- Set by the Schwarzschild radius:

$$R > 1.5 R_{\text{S}} = \left(\frac{3 G M}{c^2} \right) = 6.2 \text{ km} \left(\frac{M}{1.4 M_{\odot}} \right)$$



Subrahmanyan Chandrasekhar (born 1910)
Source: Chandra X-ray Observatory

Measured masses



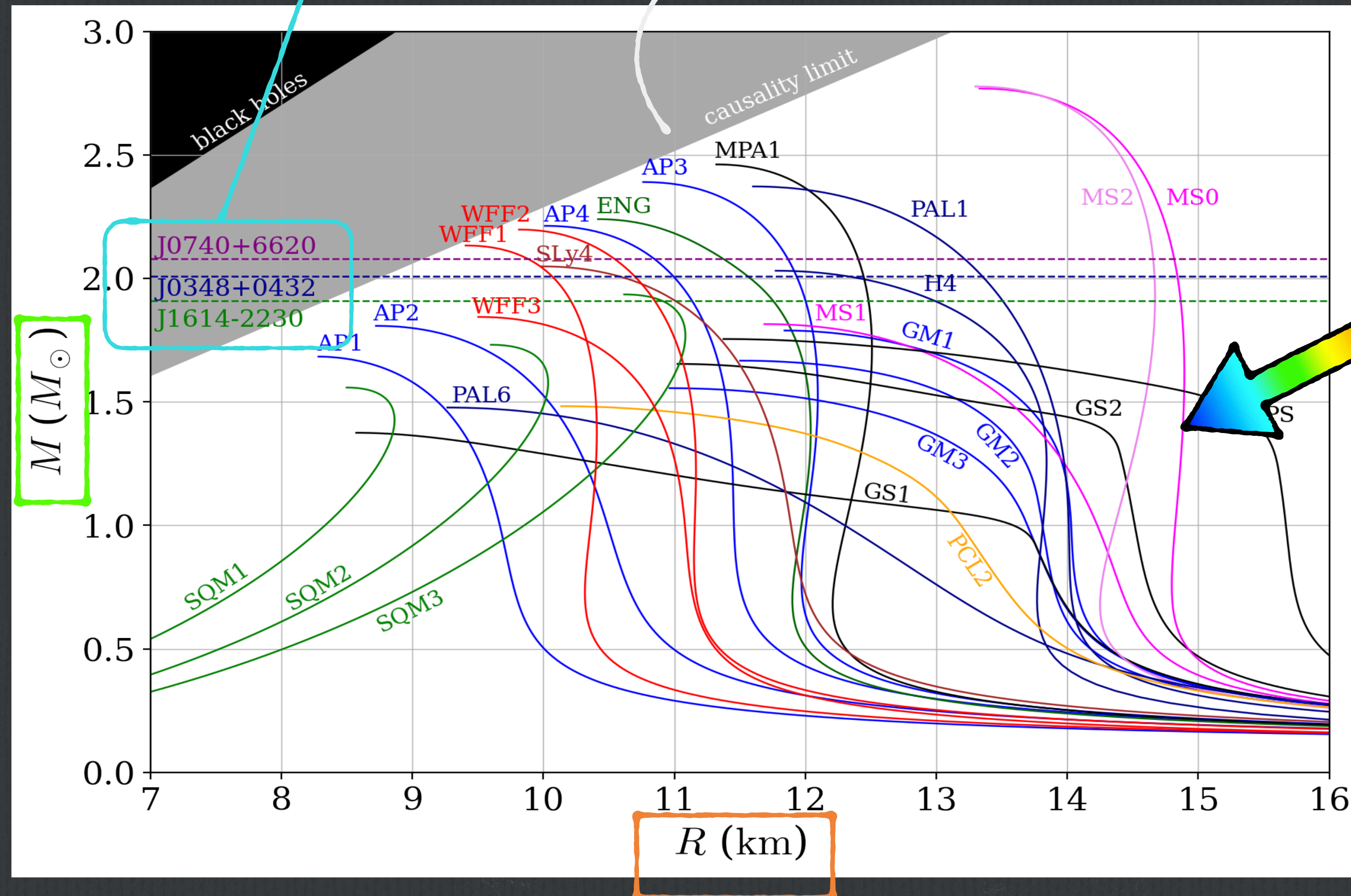
- The masses of pulsars in binary systems can be measured with high degree of accuracy through pulsar timing
- Wide distribution of masses have been measured
- Website that keeps good track of these:
https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html
- Mass measurements of heavy NS are used to constrain the EOS

Credit: Venkatraman Krishnan

Mass measurements of these 3 pulsars have already ruled out EOS models that lie beneath these measured mass values.

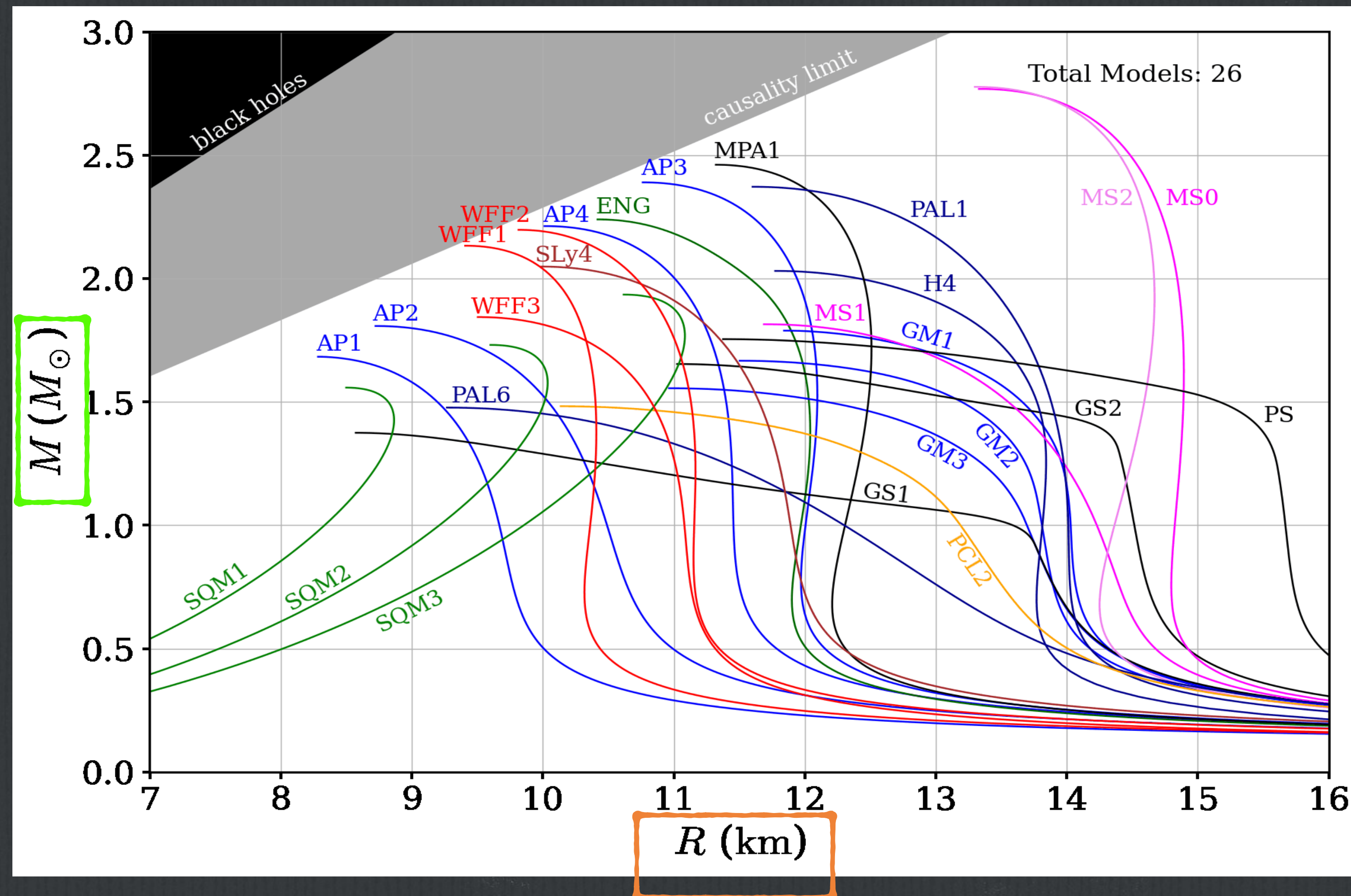
Constraining the EOS

Causality limit: the speed of sound in dense matter is less than the speed of light.



- The NS “equation of state” (EOS) aims to relate the density, mass and radius of neutron stars in a mathematical description
- No single conclusive EOS yet, but many models

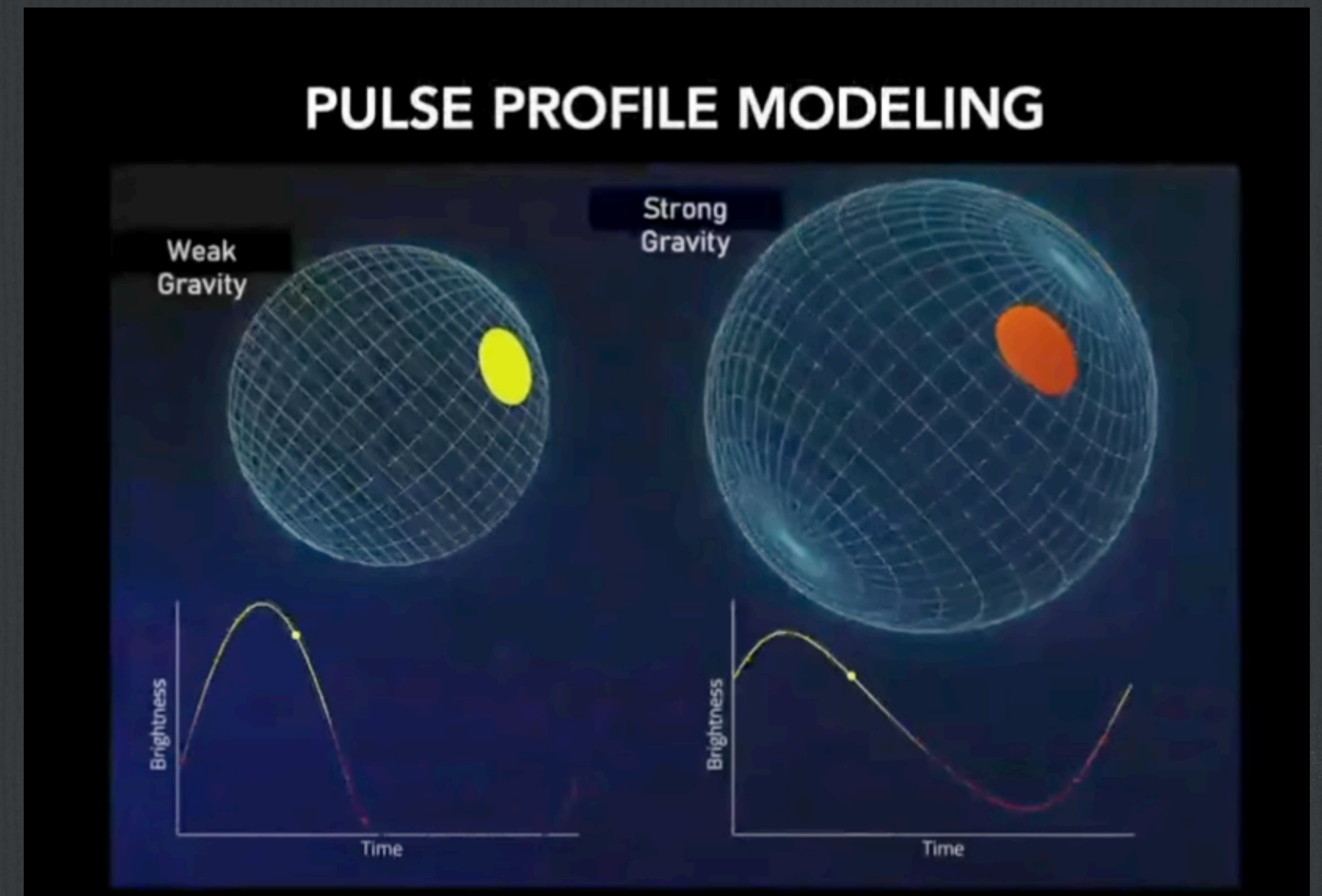
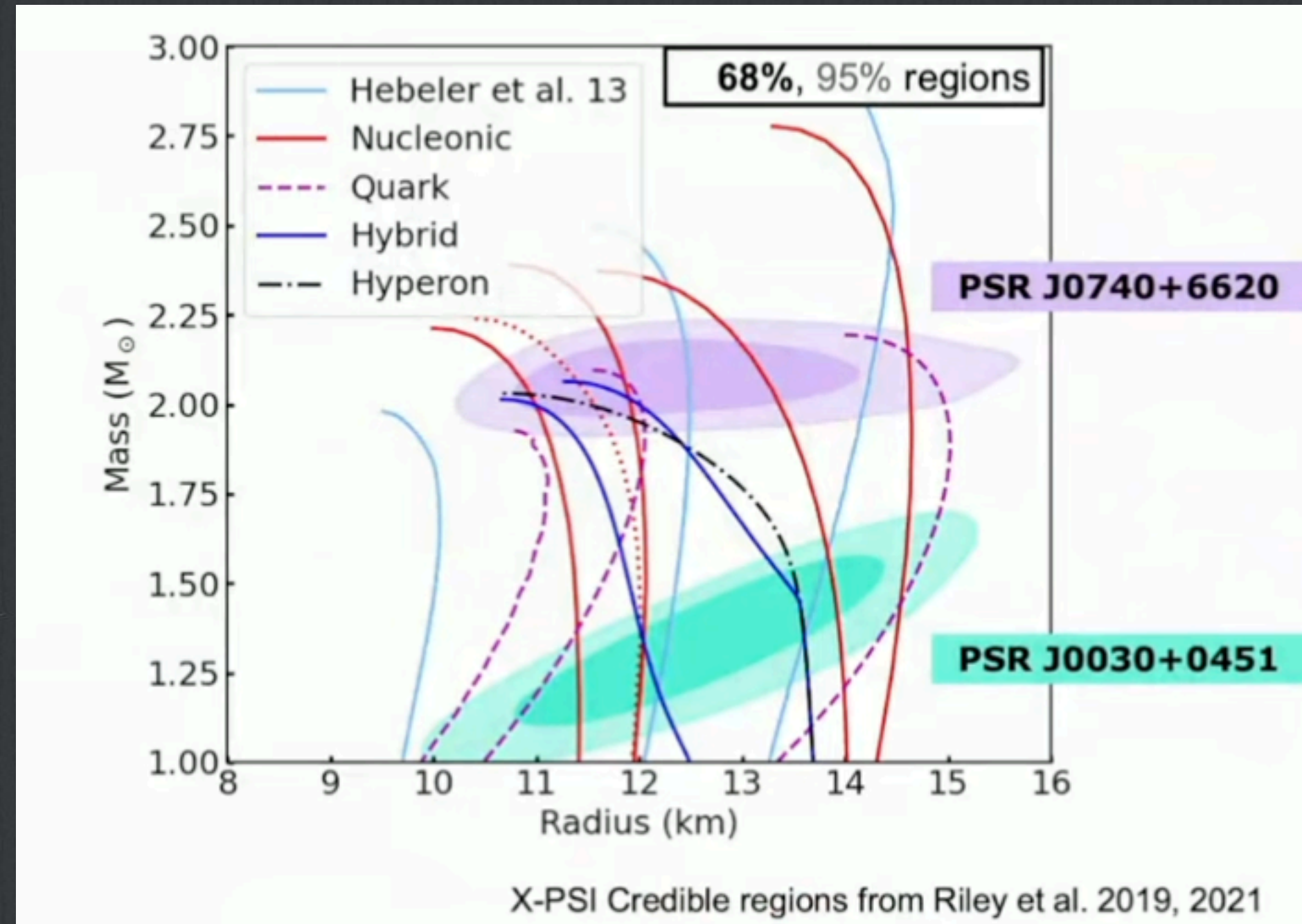
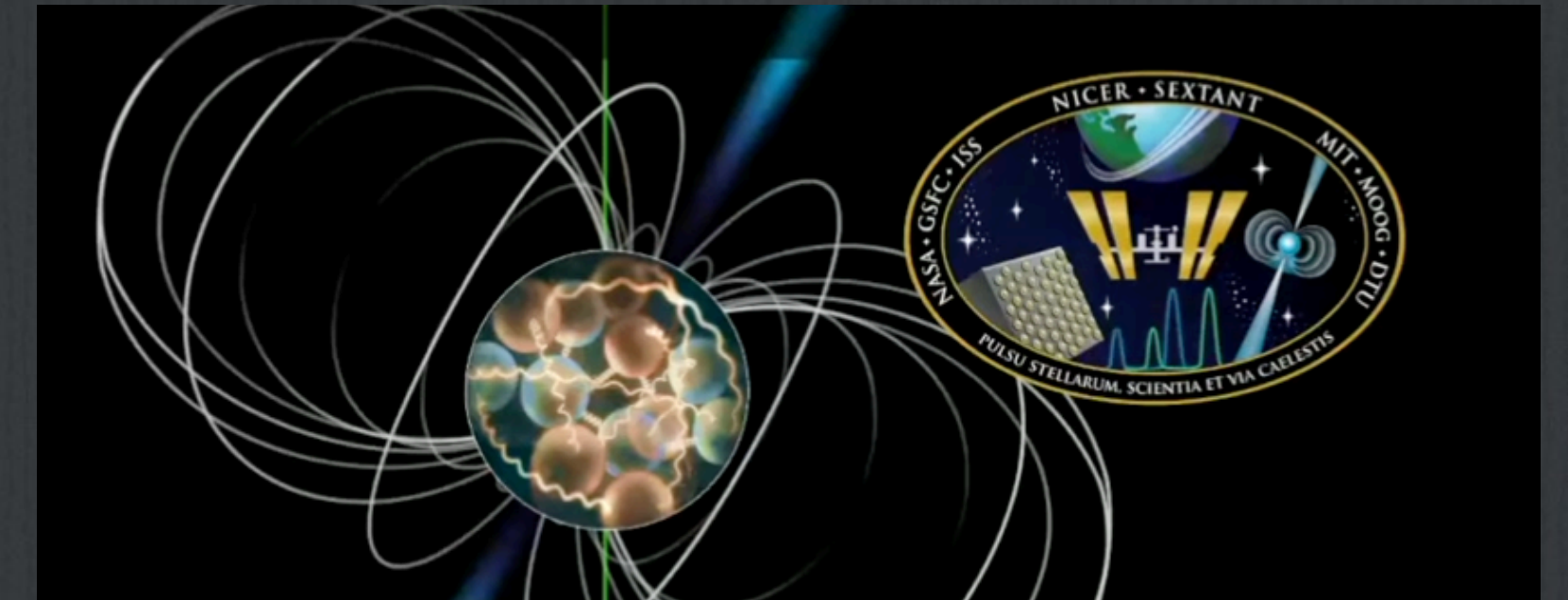
Constraining the EOS



Animation created from figures by Norbert Wex

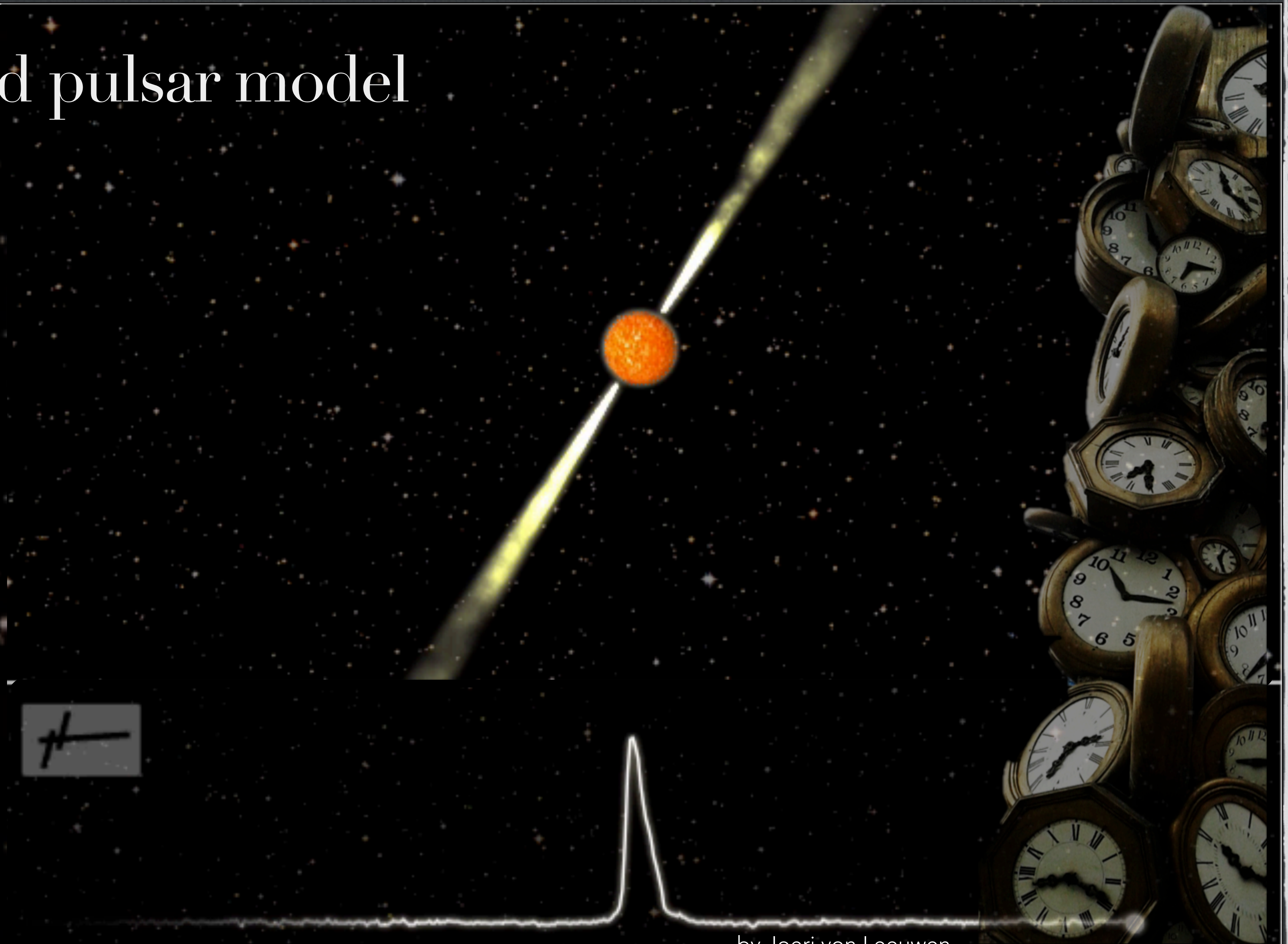
- The NS “equation of state” (EOS) aims to relate the density, **mass** and **radius** of neutron stars in a mathematical description
- No single conclusive EOS yet, but many models
- With each new NS mass observation one more theoretical model could potentially be ruled out, leaving only a last few standing
- This is partly why pulsar astronomers are so keen on accurate new massive NS mass measurements

Pinning down the radius - NICER The Neutron Star Interior Composition ExploreR



Anna Watts (Nov 2023) | <https://www.youtube.com/watch?v=shRtAEE47fE>

Pulsars: Standard pulsar model

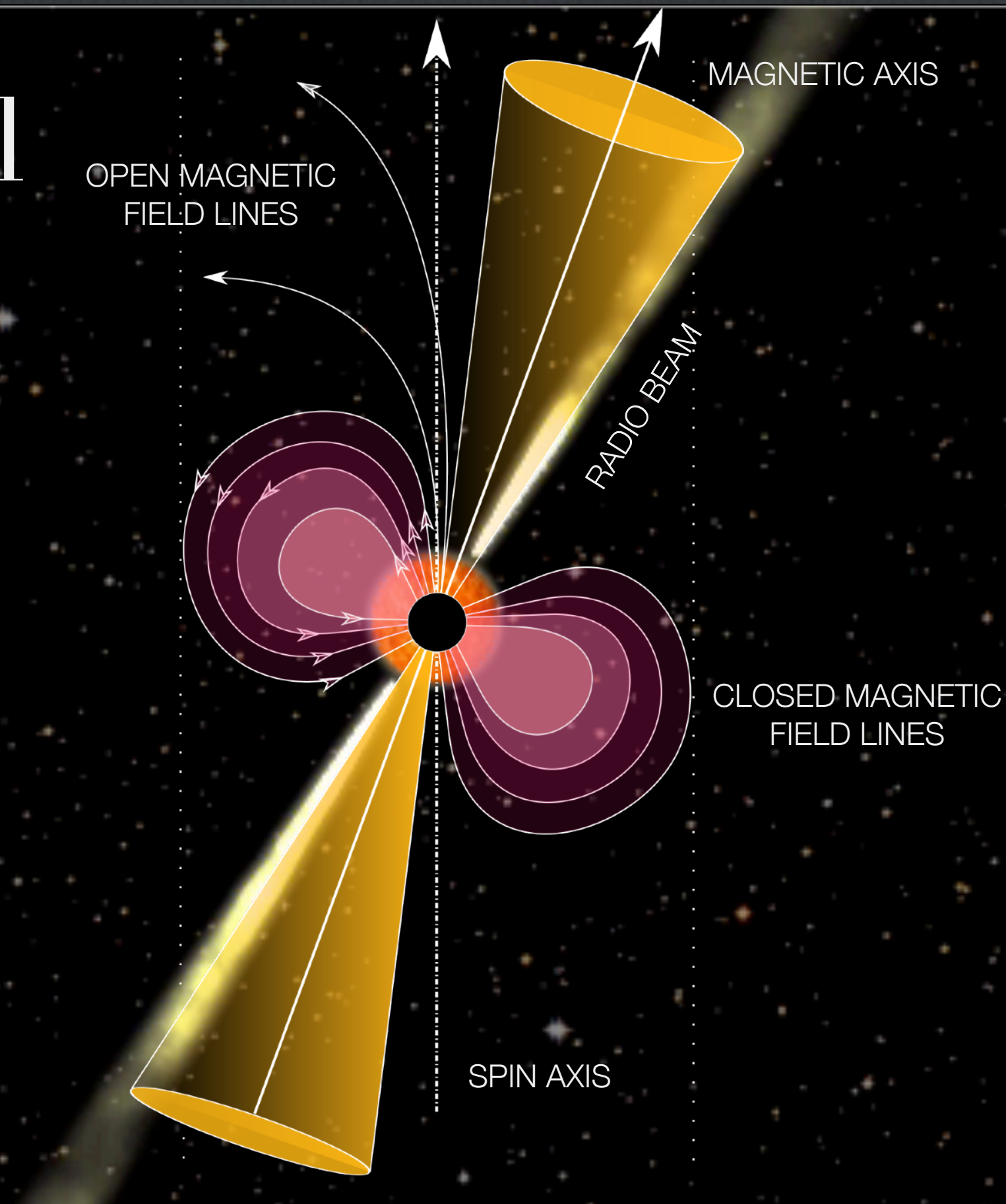


by Joeri von Leeuwen

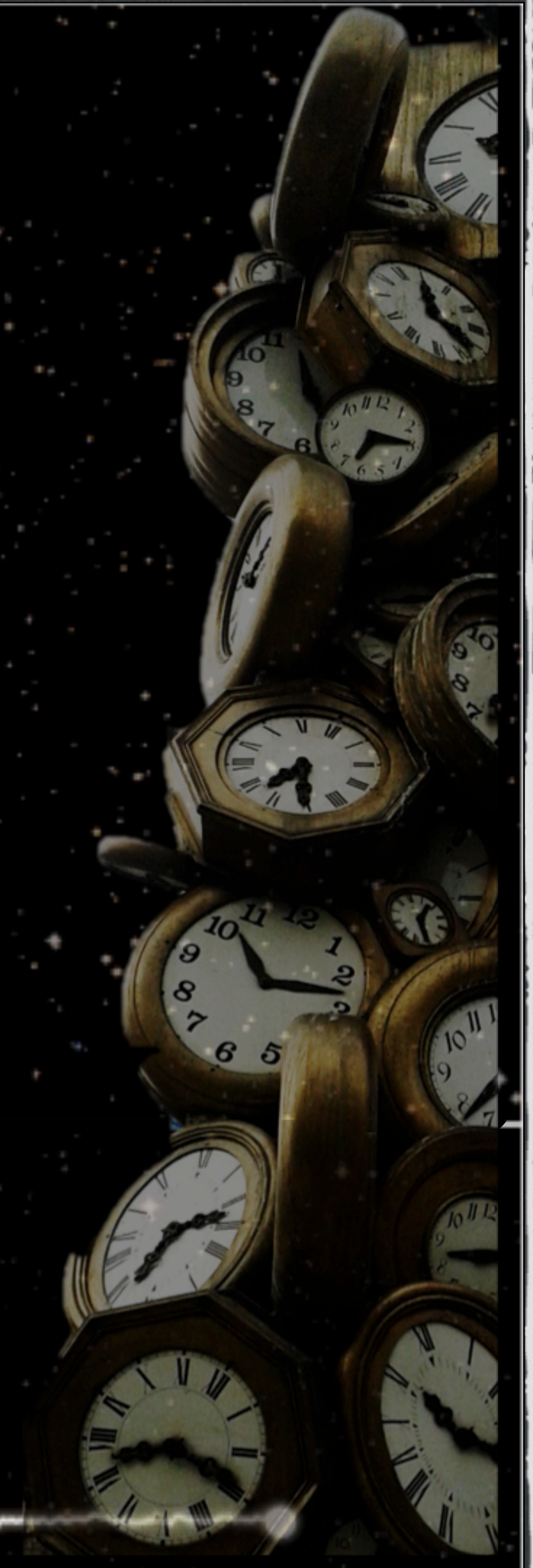
Pulsars: Standard pulsar model

Beamed neutron stars are observed as pulsars

- dead stellar cores - but not observationally dead
- radio beams emitted from magnetic poles
- with the correct alignment, these sweep our line to produce a pulse
- We can only see the pulsars for which the emission cuts our line of sight. This is greatly dependent on the orientation of the pulsar in the sky.
- We estimate that approximately 10% of all pulsars are beamed in Earth's direction and therefore potentially observable to us.



by Joeri von Leeuwen



Neutron star magnetic fields

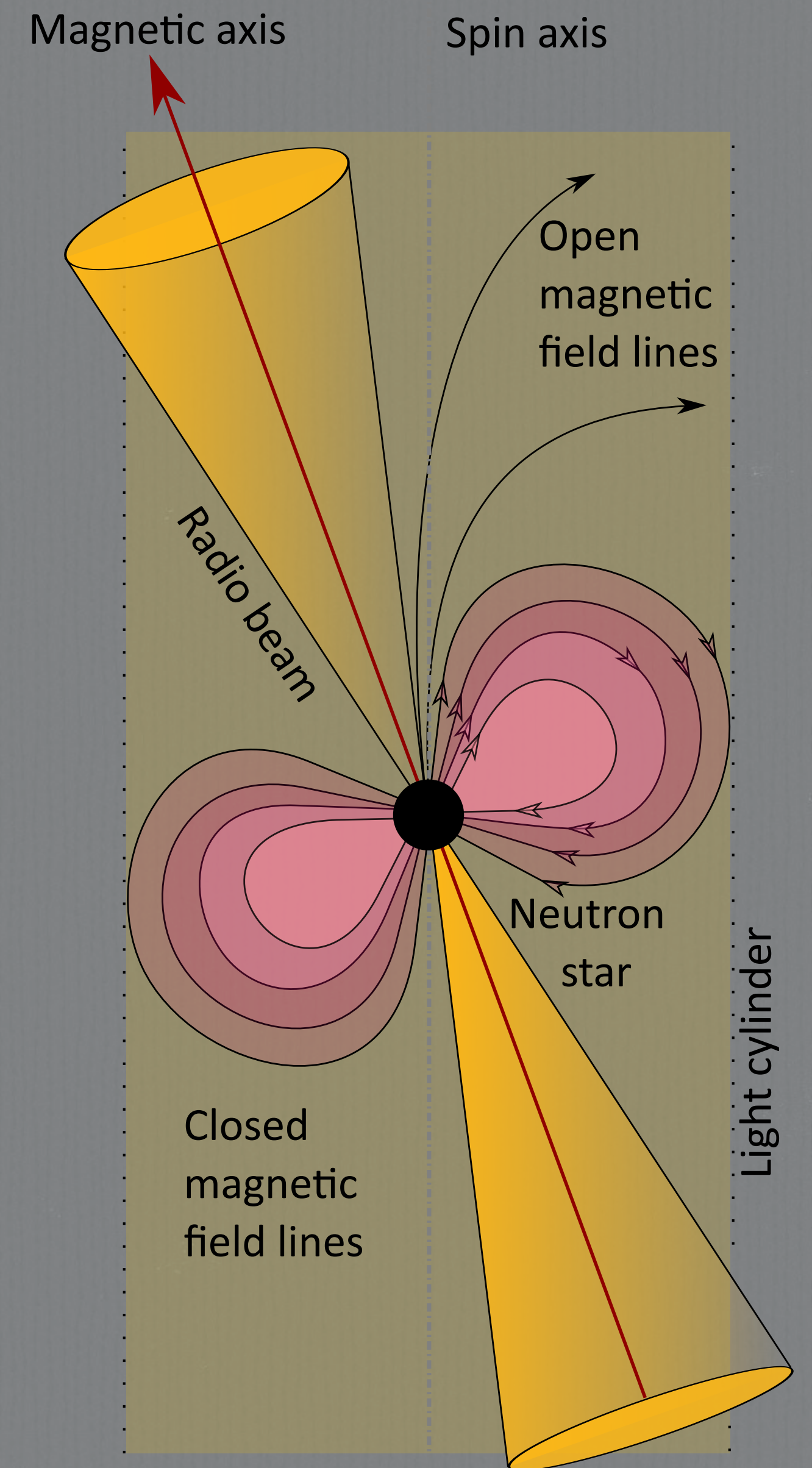
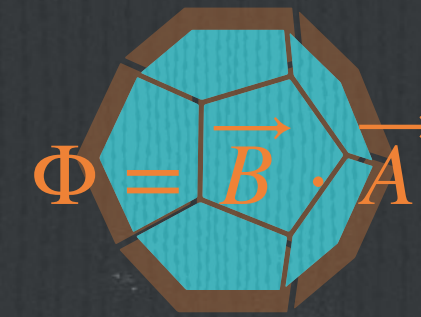
NS progenitor stars (like most stars) possess dipolar magnetic fields

Let the progenitor star have a radius $\sim 10^6$ km. When it collapses to NS with $R \sim 10$ km, it means the surface is shrinking by a factor of $\sim (10^5)^2 = 10^{10}$

During the collapse the magnetic flux is conserved, and since the magnetic field scales with surface area, that means the magnetic field increases by a factor of 10^{10} !

Q: what are typical magnetic fields of massive stars?

Φ - Magnetic flux
 $\vec{B}$ - Magnetic field
 $\vec{A}$ - Area



Neutron star magnetic fields

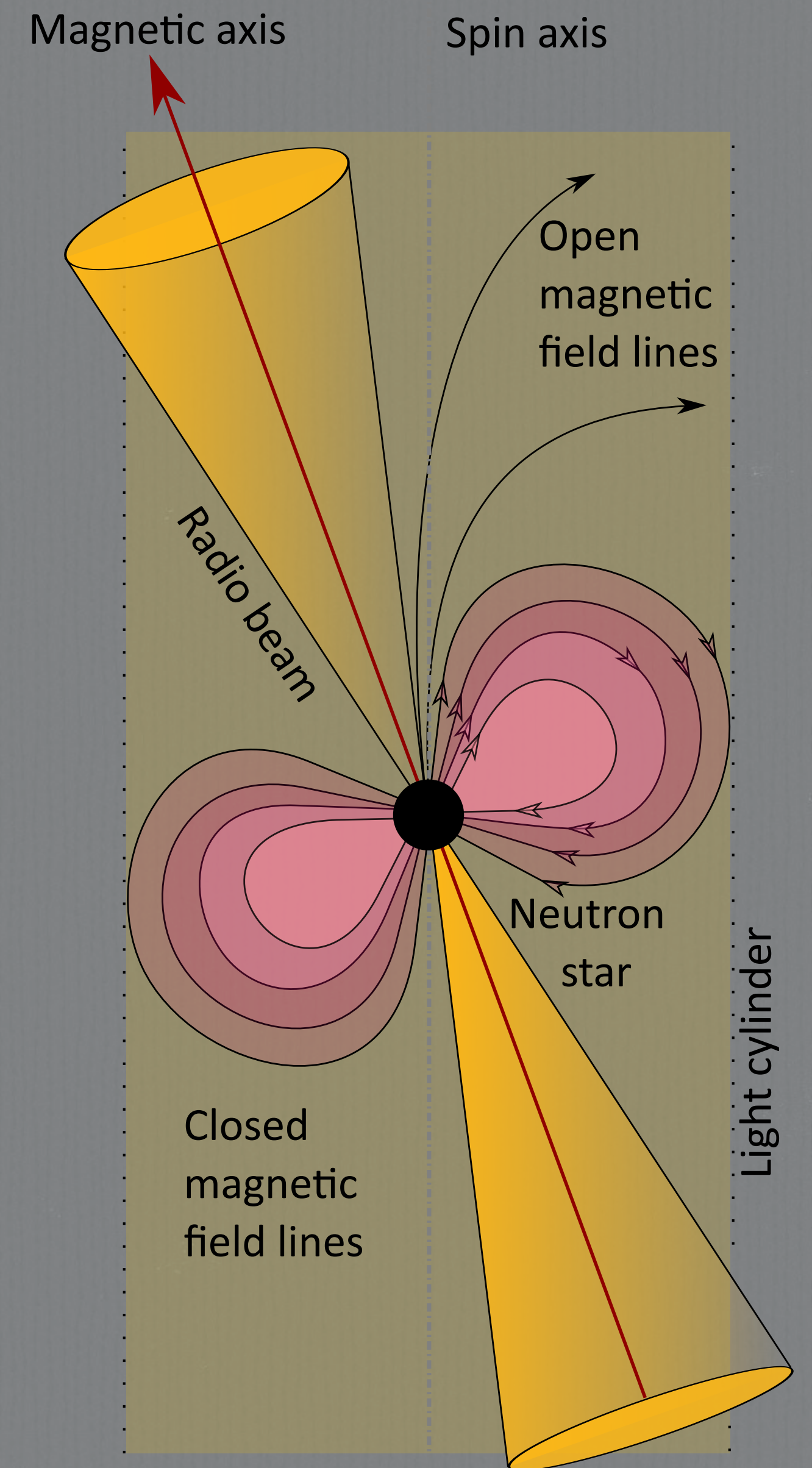
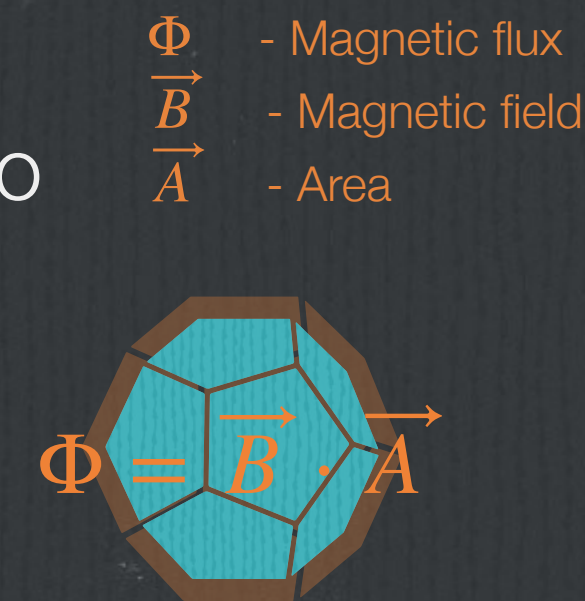
NS progenitor stars (like most stars) possess dipolar magnetic fields

Let the progenitor star have a radius $\sim 10^6$ km. When it collapses to NS with $R \sim 10$ km, it means the surface is shrinking by a factor of $\sim (10^5)^2 = 10^{10}$

During the collapse the magnetic flux is conserved, and since the magnetic field scales with surface area, that means the magnetic field increases by a factor of 10^{10} !

Q: what are typical magnetic fields of massive stars?

The initial 100 Gauss magnetic field is scaled to become a NS field of 10^{12} G during collapse.



Spinning down: magnetic dipolar radiation

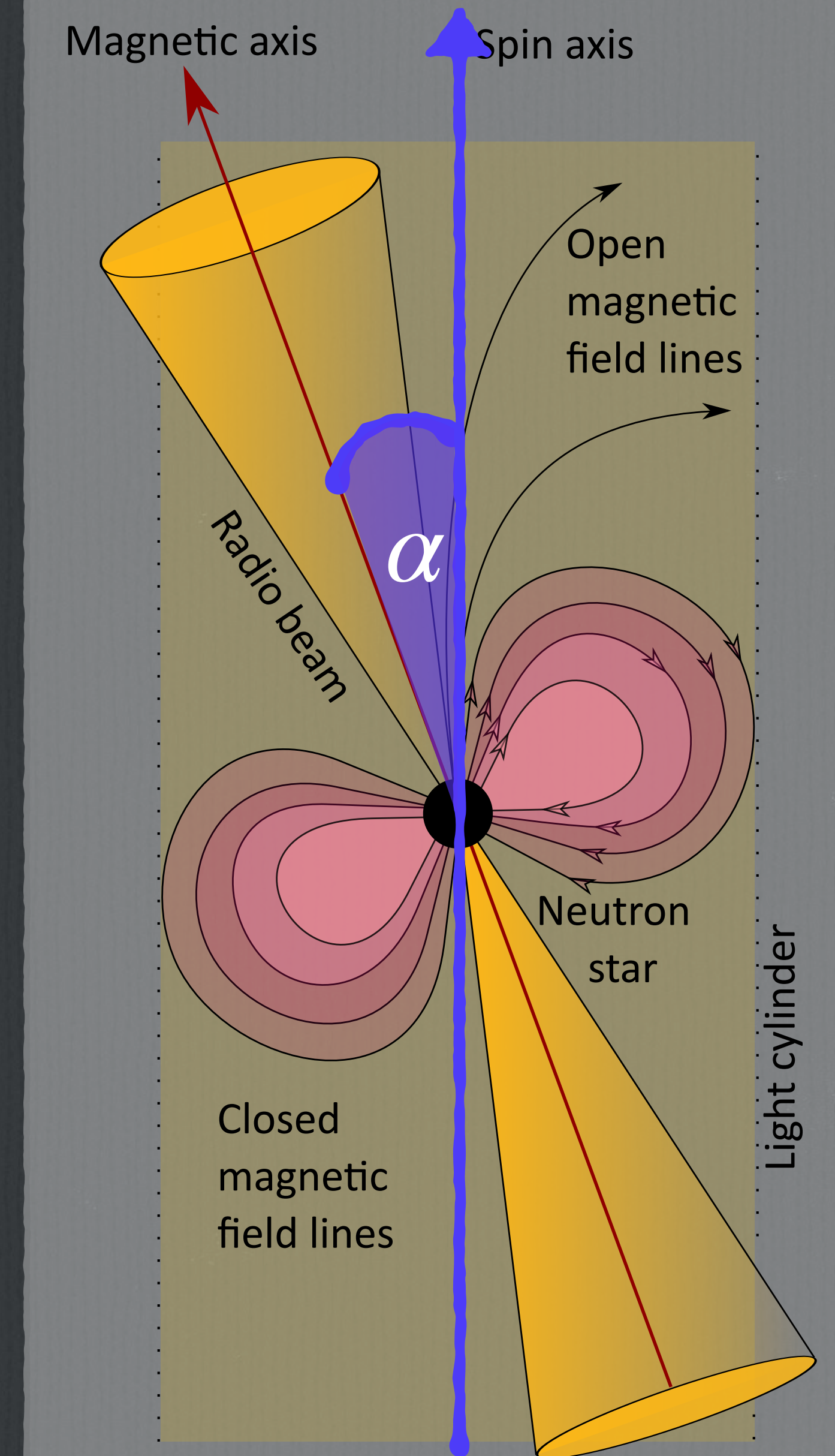
- Pulsars are observed to spin down, in other words the period P is seen to slowly increase with time

$$\dot{P} = \frac{dP}{dt} > 0$$

- What causes it to slow down?
- A rotating magnetic dipole (inclined w.r.t the spin axis) emits radiation at the rotational frequency of the system.

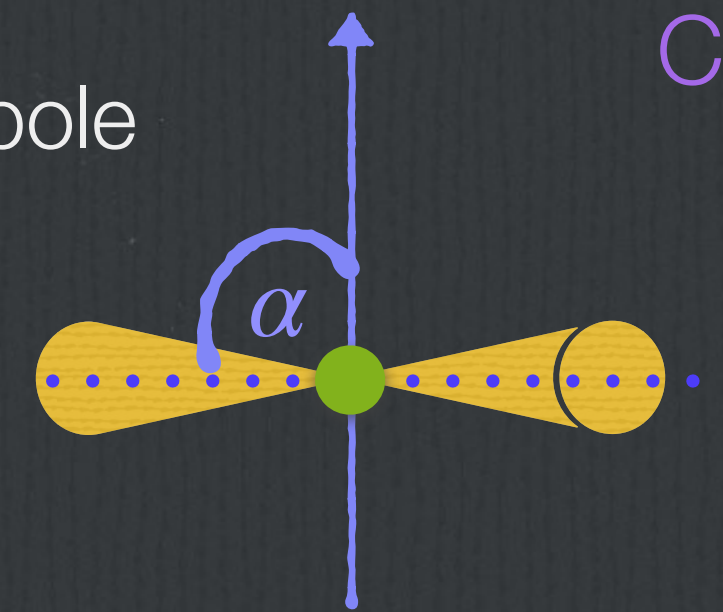
$$P_{\text{rad}} = \frac{2}{3c^3} (BR^3 \sin \alpha)^2 \left(\frac{2\pi}{P} \right)^4 \lesssim \frac{32 B^2 R^6 \pi^4}{3c^3 P^4} \quad [\text{J/s} = \text{W}]$$

This dipole radiation extracts from the rotational energy of NS causing it to slow down.



Magnetic Fields

- Assuming the loss rotational energy is entirely due to the magnetic dipole radiation, we can estimate the magnetic field of a pulsar
- And orthogonal rotator



Canonical moment of inertia

$$I = \frac{2MR^2}{5}$$

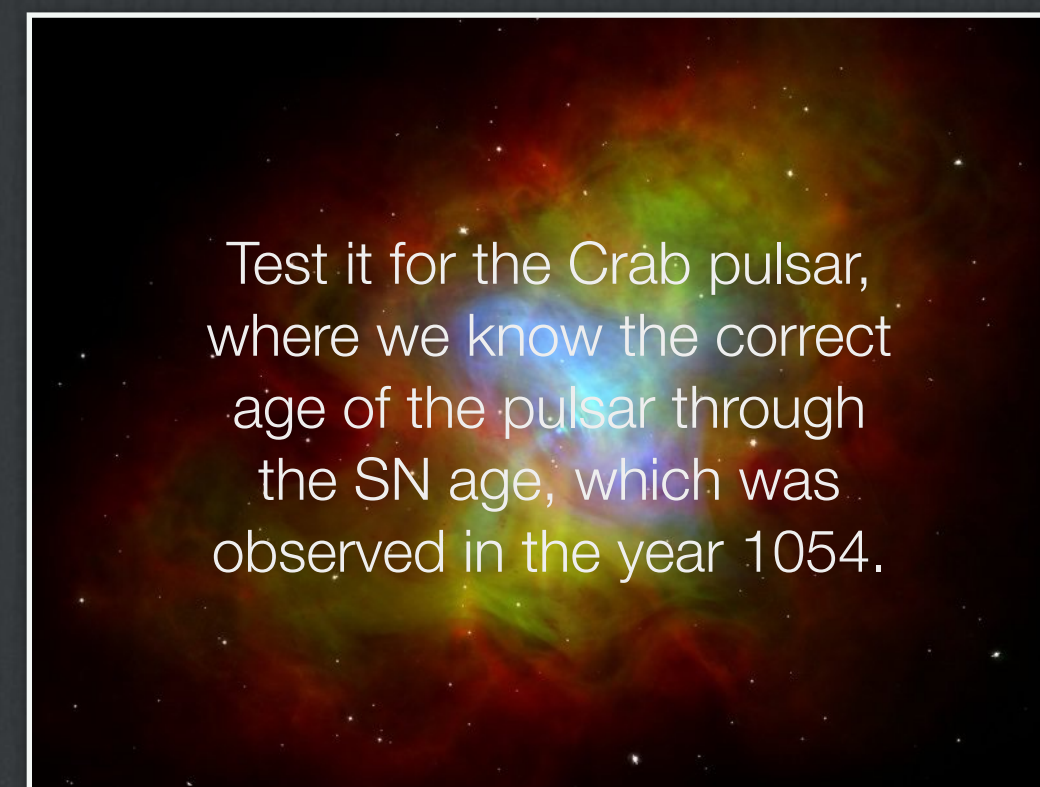
$$B = 10^8 - 10^{14} \text{ G}$$

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} \lesssim \frac{32 B^2 R^6 \pi^4}{3c^3 P^4} \Rightarrow B \gtrsim \sqrt{\frac{3 I c^3}{8 \pi^2 R^6}} \times P \dot{P}$$

Characteristic Age

$$\tau_c \equiv \frac{P}{2\dot{P}} \simeq 15.8 \text{ Myr} \left(\frac{P}{\text{s}} \right) \left(\frac{\dot{P}}{10^{-15}} \right)^{-1}$$



Test it for the Crab pulsar, where we know the correct age of the pulsar through the SN age, which was observed in the year 1054.

Characteristic Age

$$\frac{P}{2\dot{P}} = 3.93 \times 10^{10} \text{ s} \sim 1250 \text{ yr}$$

True Age

$$(2026 - 1054) \text{ yr} = 972 \text{ yr}$$

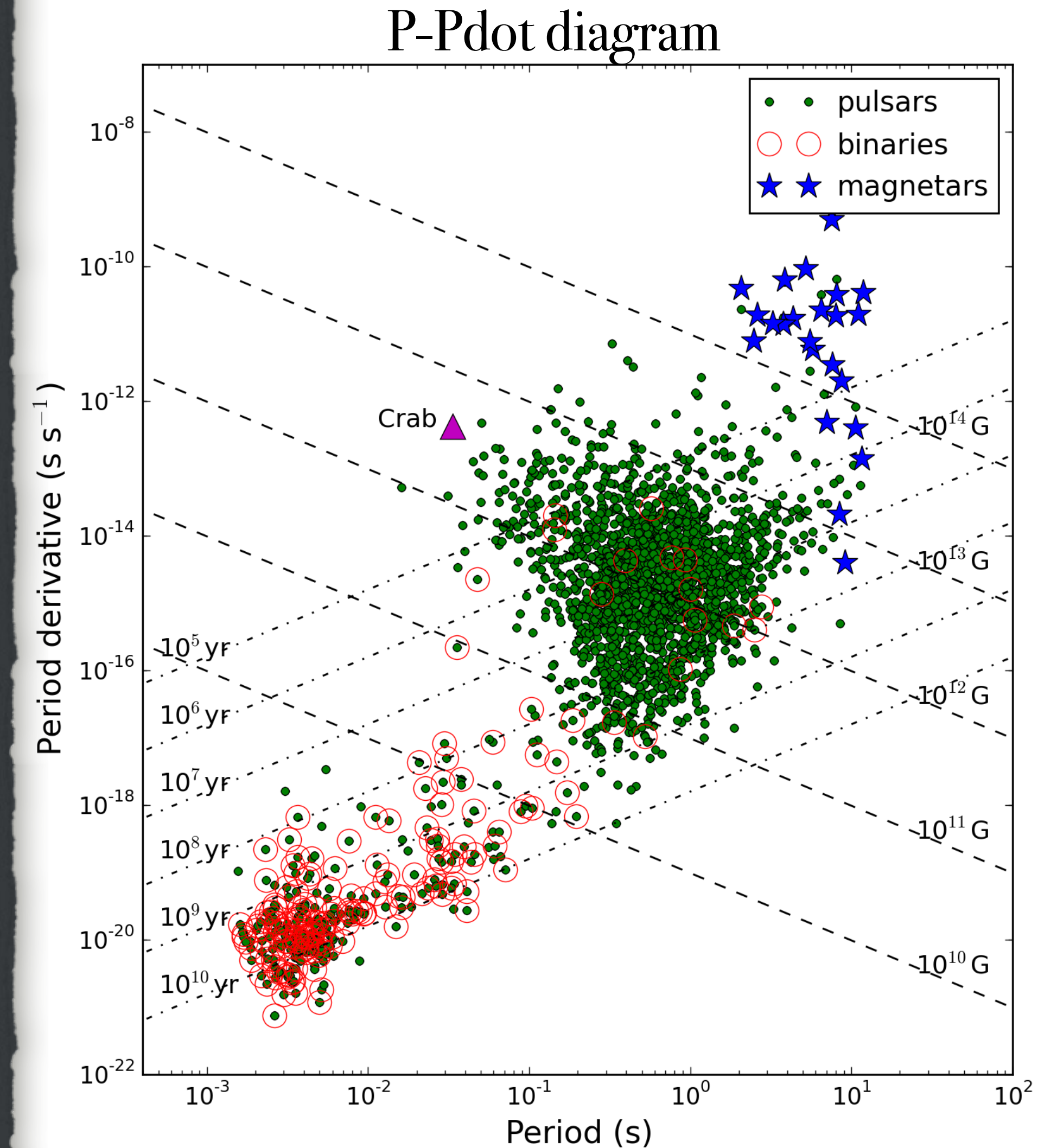
Adding P , $P\text{-dot}$, magnetic field and characteristic age to one diagram!

The Radio Pulsar population

- Over 3000 pulsars discovered to date, most of which reside in the MW
MeerKAT has already observed > 1500 pulsars!

Q: Do we see them outside the MW? (Answer in Lecture 2)

slow down faster



spin slower

The Radio Pulsar population

- Over 4000 pulsars discovered to date, most of which reside in the MW

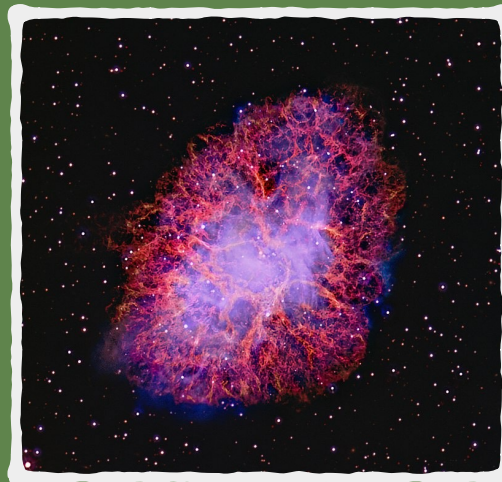
‘Normal or isolated’ pulsars ($P > 0.1$ sec)

pulsars are born in SNR spinning fast

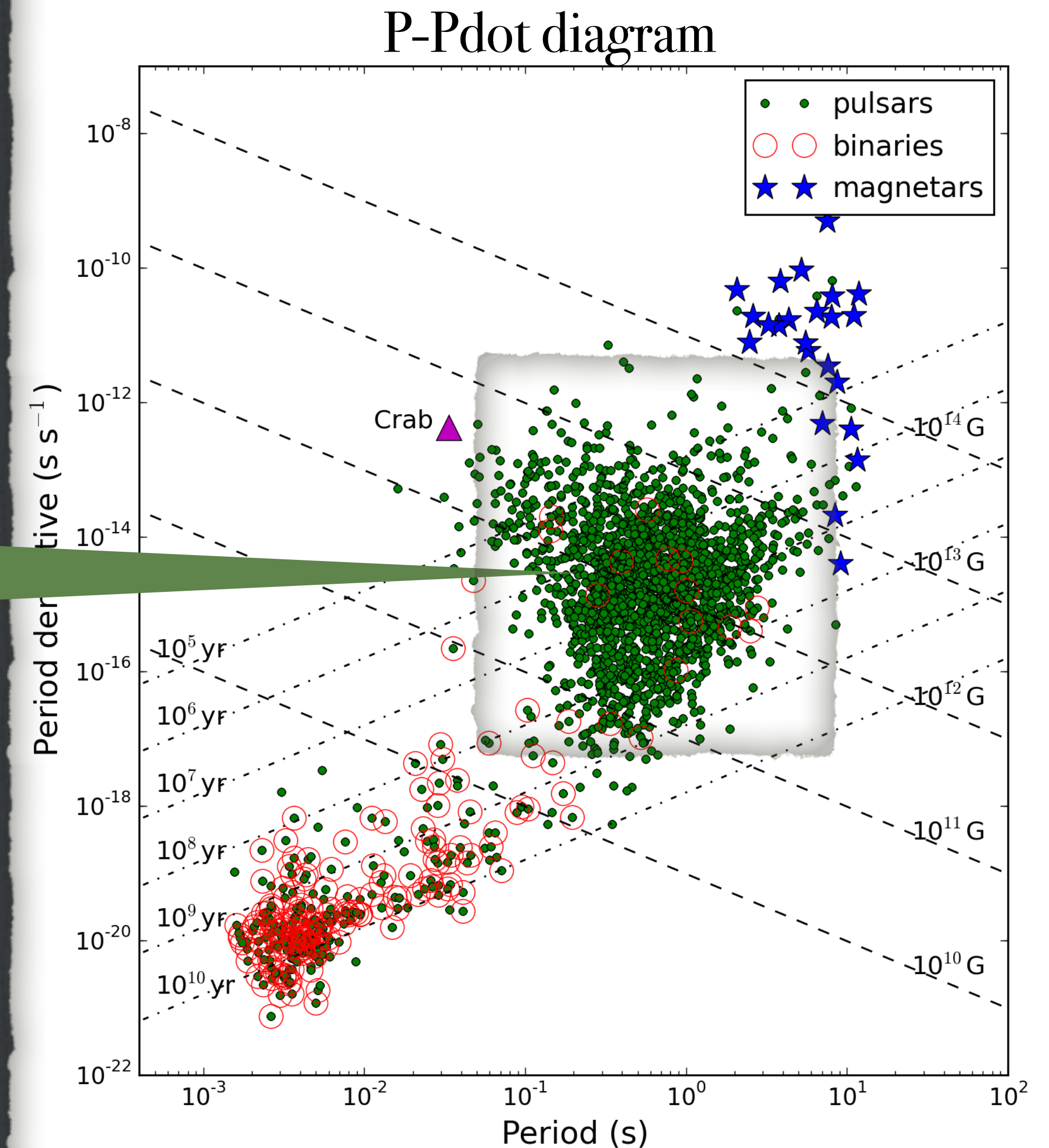
over the course of their lives they spin down

when the spin rate is too slow to produce radio emission, they shut off

In the computation of characteristic age we assumed the B-field stays constant, that's why the “age arrow” is along a constant B-field, but across the characteristic age lines.



slow down faster



The Radio Pulsar population

- Over 4000 pulsars discovered to date, most of which reside in the MW

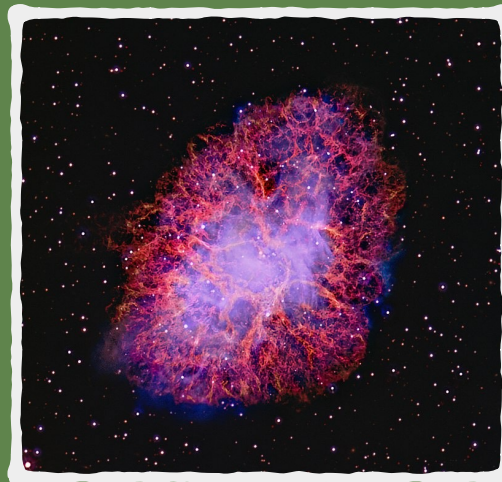
‘Normal or isolated’ pulsars ($P > 0.1$ sec)

pulsars are born in SNR spinning fast

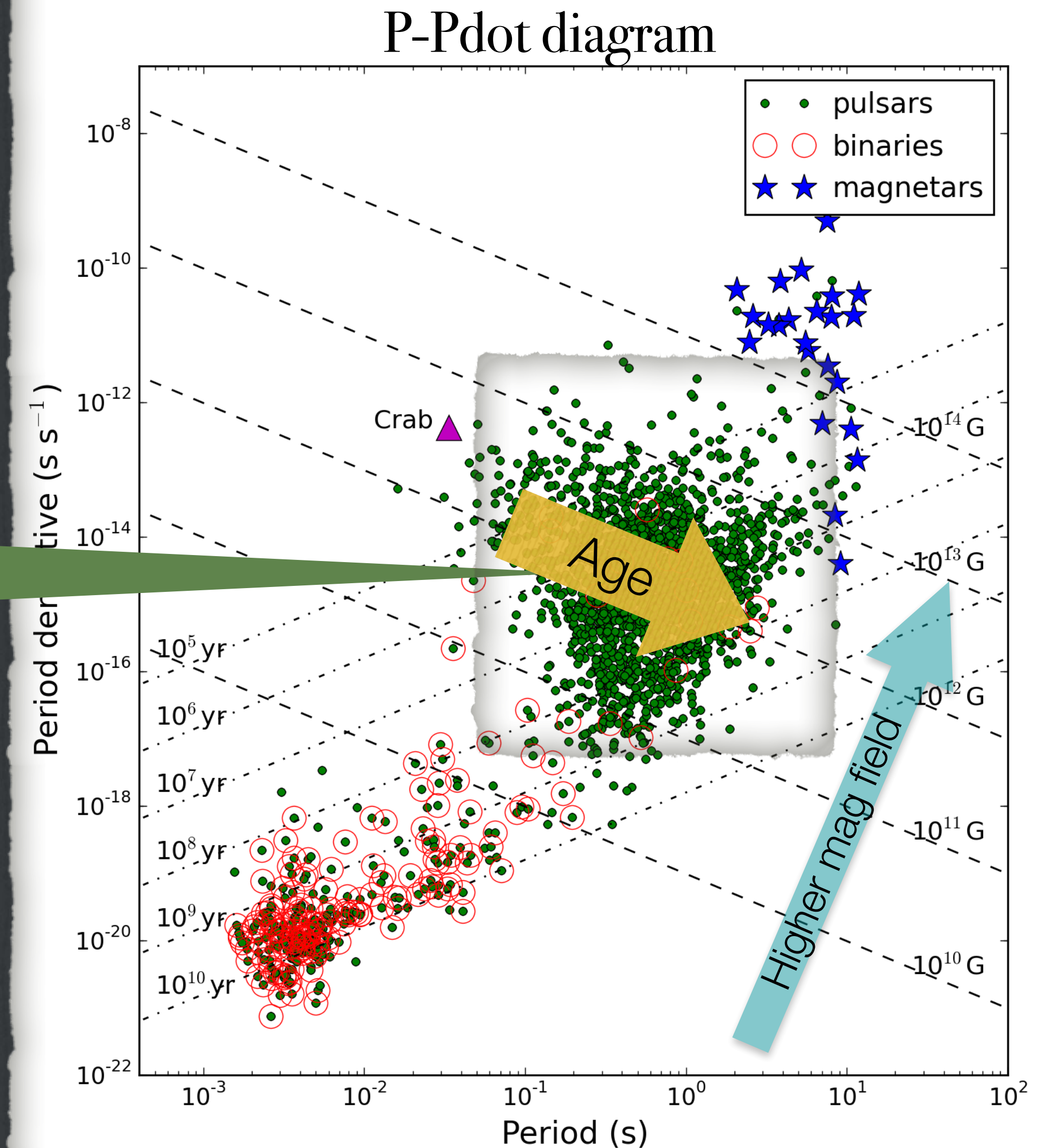
over the course of their lives they spin down

when the spin rate is too slow to produce radio emission, they shut off

In the computation of characteristic age we assumed the B-field stays constant, that's why the “age arrow” is along a constant B-field, but across the characteristic age lines.



slow down faster



spin slower

The Radio Pulsar population

- Over 4000 pulsars discovered to date, most of which reside in the MW

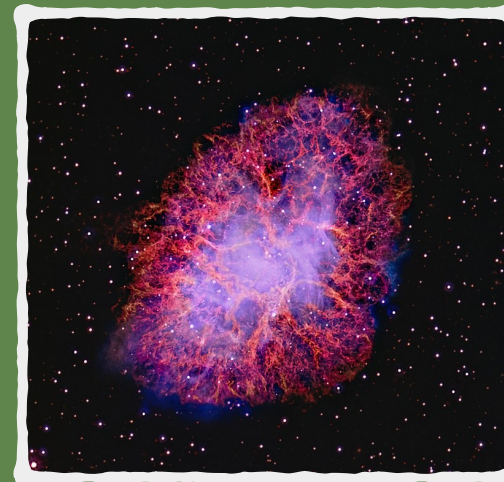
'Normal or isolated' pulsars ($P > 0.1$ sec)

pulsars are born in SNR spinning fast

over the course of their lives they spin down

when the spin rate is too slow to produce radio emission, they shut off

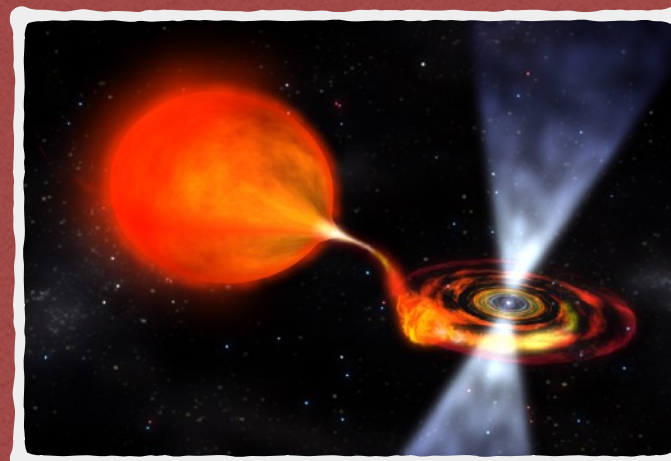
In the computation of characteristic age we assumed the B-field stays constant, that's why the "age arrow" is along a constant B-field, but across the characteristic age lines.



Millisecond pulsars

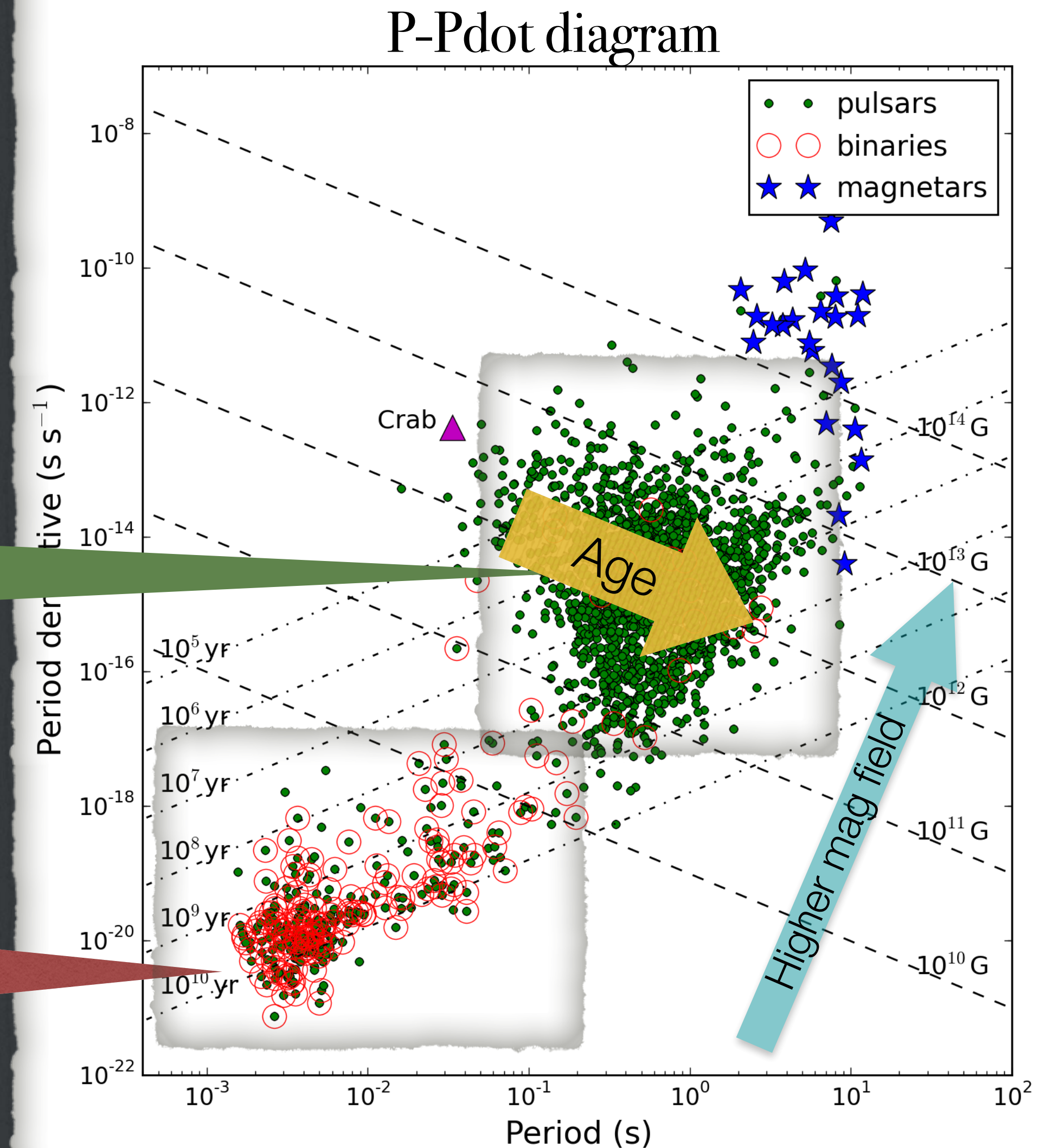
Approx. 10% of the pulsar population

MSPs are most often in binary systems, and have been spun up through accretion to incredible rotations of a few ms!



MSPs are used for high precision pulsar timing experiments

slow down faster



The Radio Pulsar population

- Over 4000 pulsars discovered to date, most of which reside in the MW

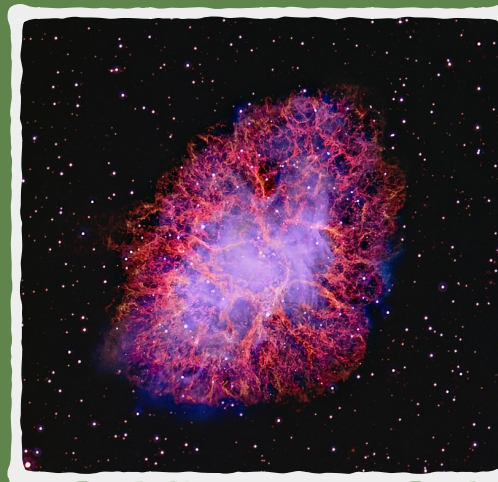
'Normal or isolated' pulsars ($P > 0.1$ sec)

pulsars are born in SNR spinning fast

over the course of their lives they spin down

when the spin rate is too slow to produce radio emission, they shut off

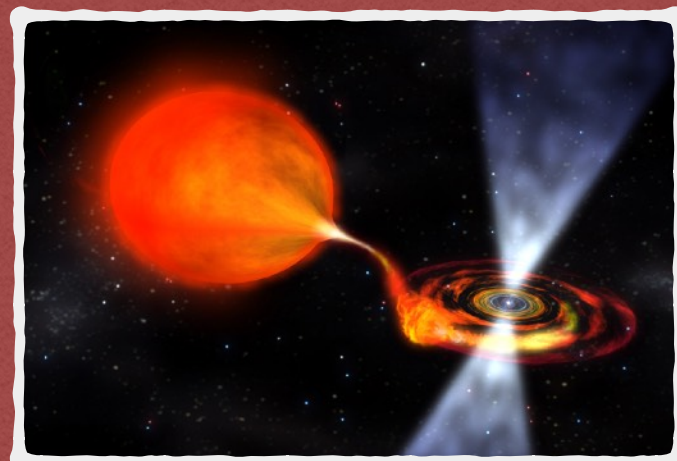
In the computation of characteristic age we assumed the B-field stays constant, that's why the "age arrow" is along a constant B-field, but across the characteristic age lines.



Millisecond pulsars

Approx. 10% of the pulsar population

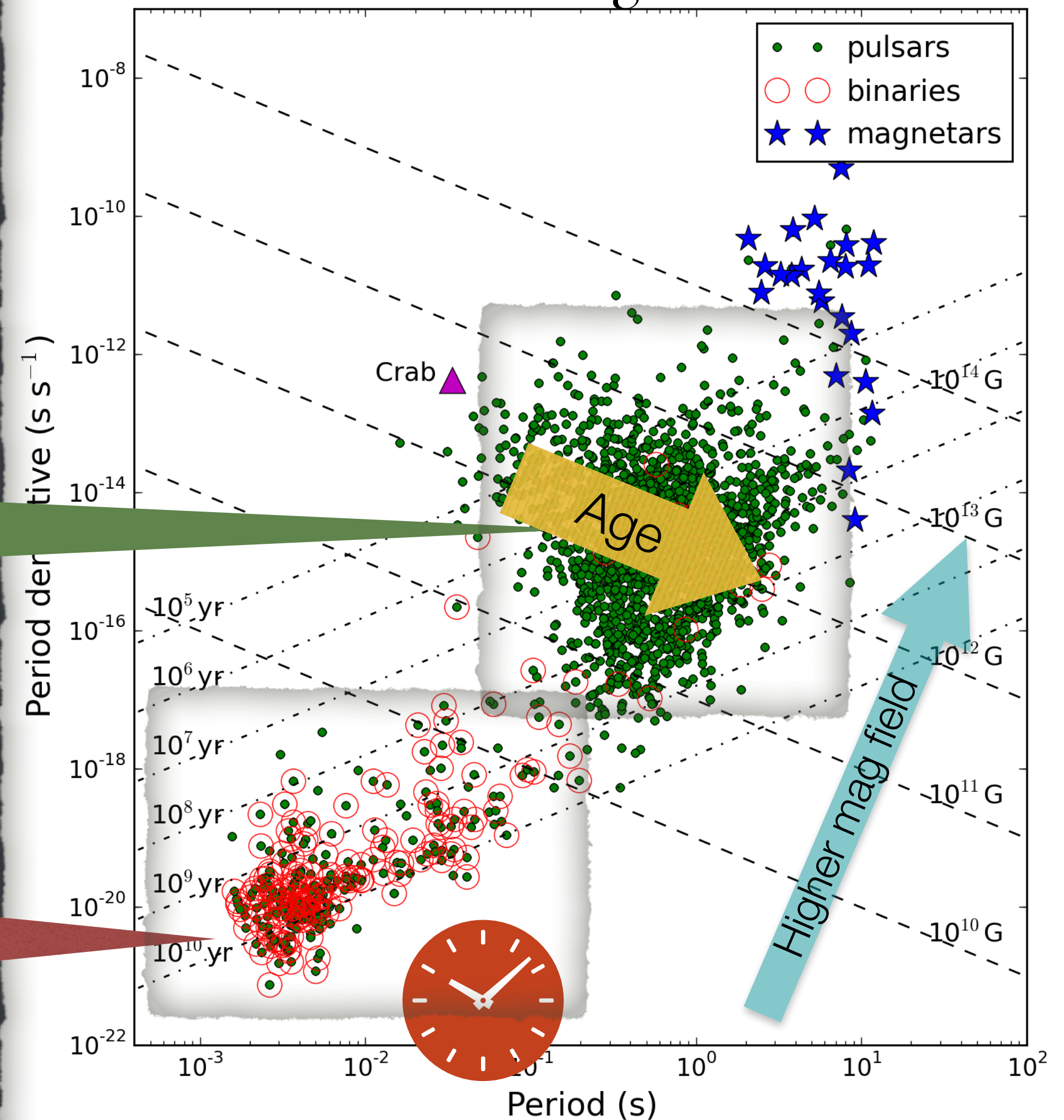
MSPs are most often in binary systems, and have been spun up through accretion to incredible rotations of a few ms!



MSPs are used for high precision pulsar timing experiments

slow down faster

P-Pdot diagram



spin slower

The Evolution of Millisecond Pulsars (MSPs)

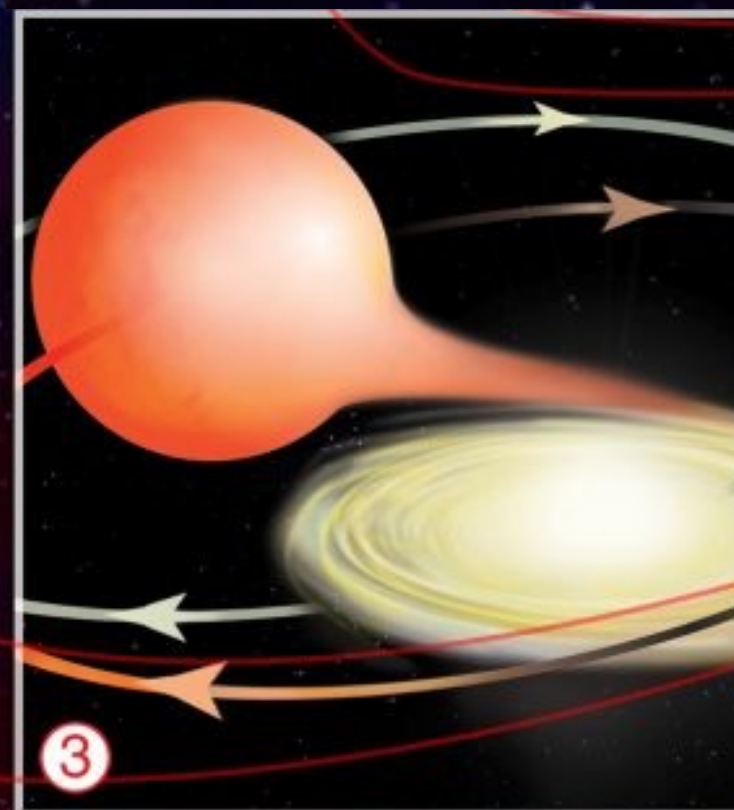
1. Binary of a massive star and a "normal" Sun-like star



3. After billions of years, the lower-mass star evolves and expands into a red giant.

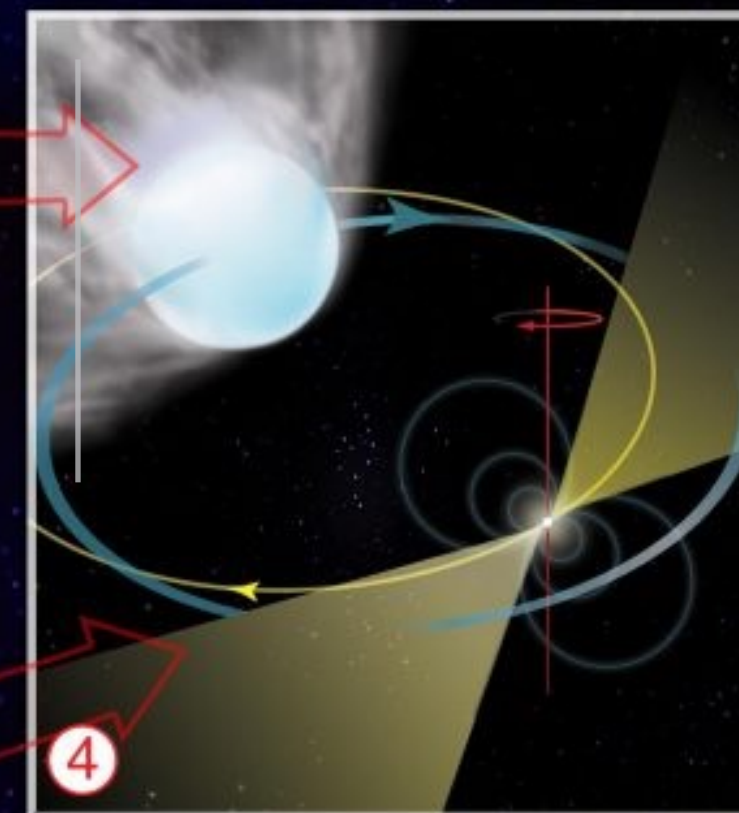
The strong gravitational field from the dense NS strips material from the red giant companion through the process of accretion. Leading to a disk of accreting material around the NS and eventually onto the NS.

This transfer of angular momentum spins up the NS, making it spin more rapidly.



4. Once accretion ends, the NS is spinning very rapidly ($P < 30\text{ms}$) and is again emitting beamed radio emission as a MSP.

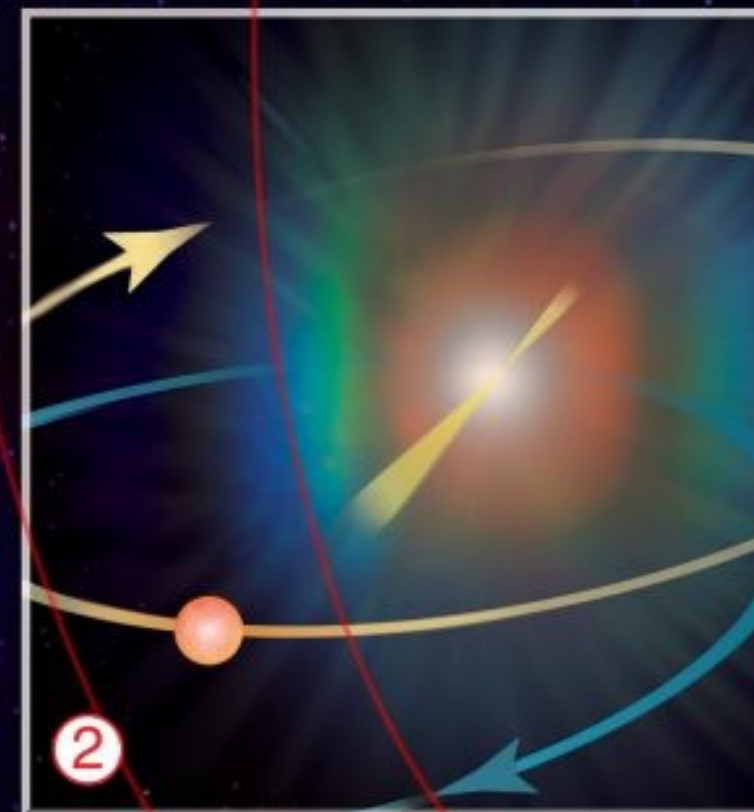
Because the NS has been switched on for a second time - the process is also called 'recycling'.



2. The massive star exhausts its nuclear fuel and dies in a SN leaving a NS as its remnant.

For several 10 millions of years, that NS is likely active as a radio pulsar.

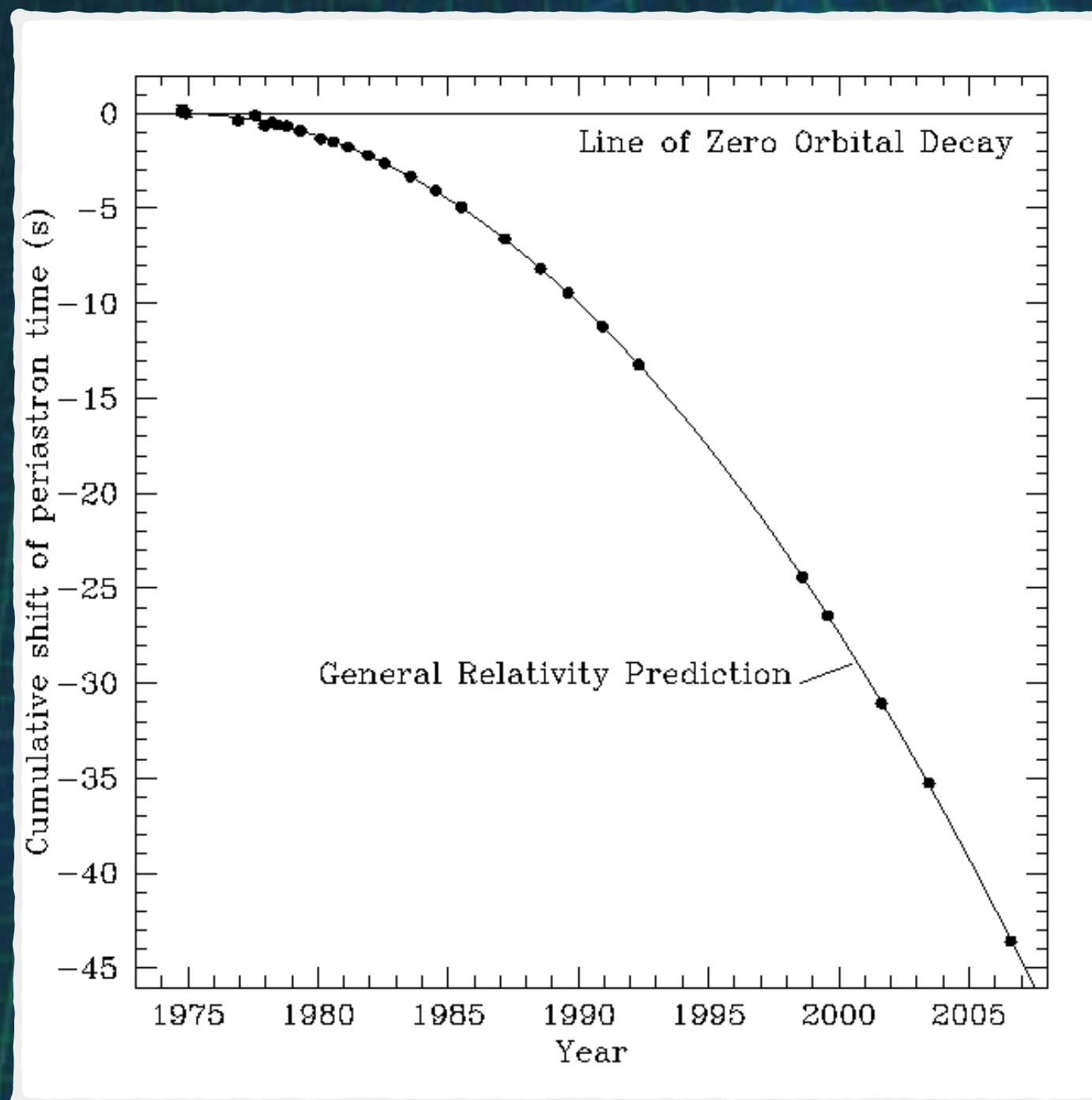
Eventually it slows down, turns off and becomes a cooling NS



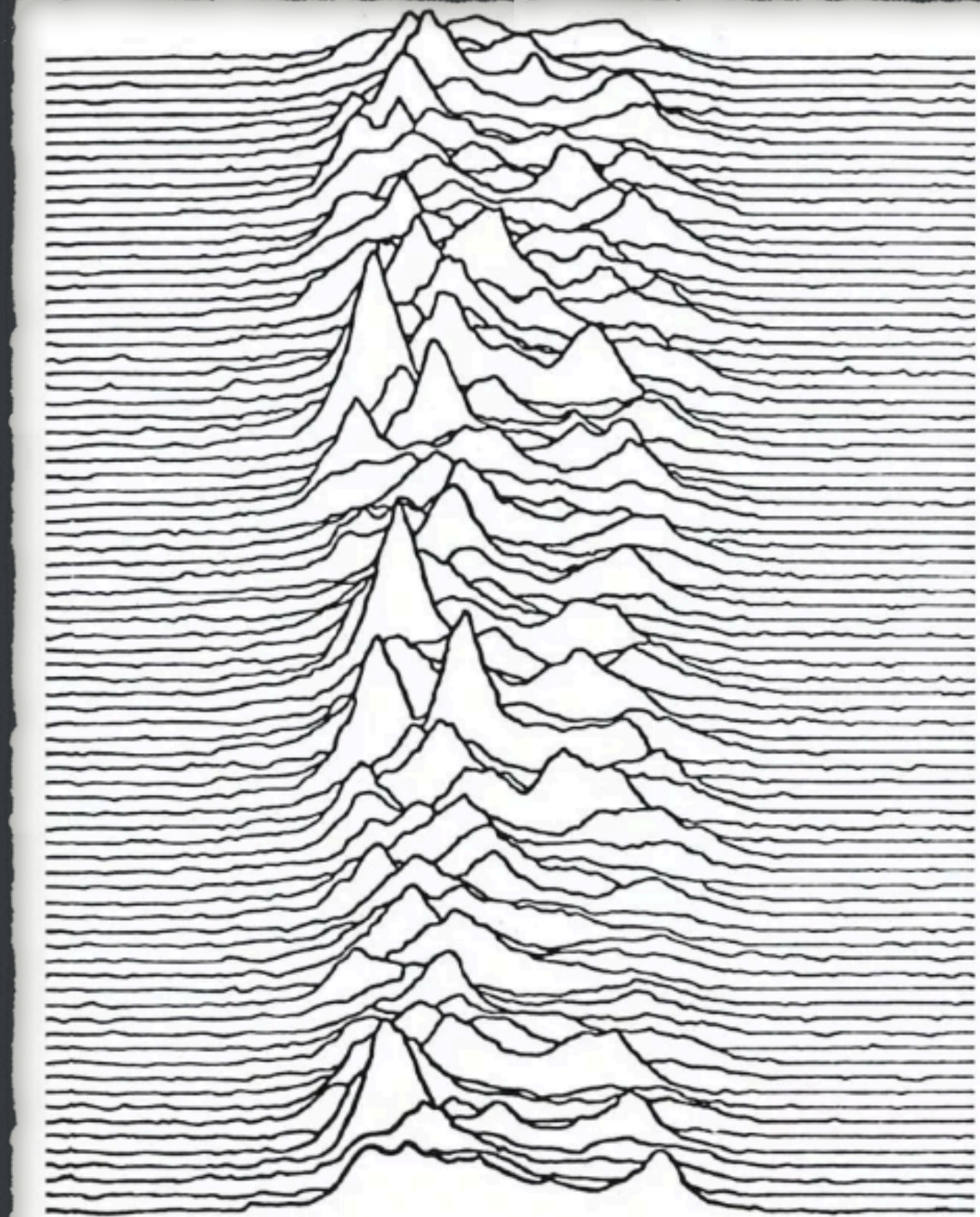
First Binary: Orbital decay due to GW emission

Hulse-Taylor PSR B1913+16

- First binary pulsar, discovered by Russell Hulse and Joseph Taylor in 1974
- Relativistic binary of NS and pulsar in orbit
- Pulsar: 59 ms pulse period
- Orbit: **7.75 hr** orbit
- Orbital precession: $d\omega/dt = 4.2^\circ/\text{yr}$



- Orbital decay due to GW emission
- NSs are getting closer
- Semimajor axis decrease 3.5 m/yr
- Inspiral in 300 million yrs



Part 2: Pulsar observations and Data Analysis

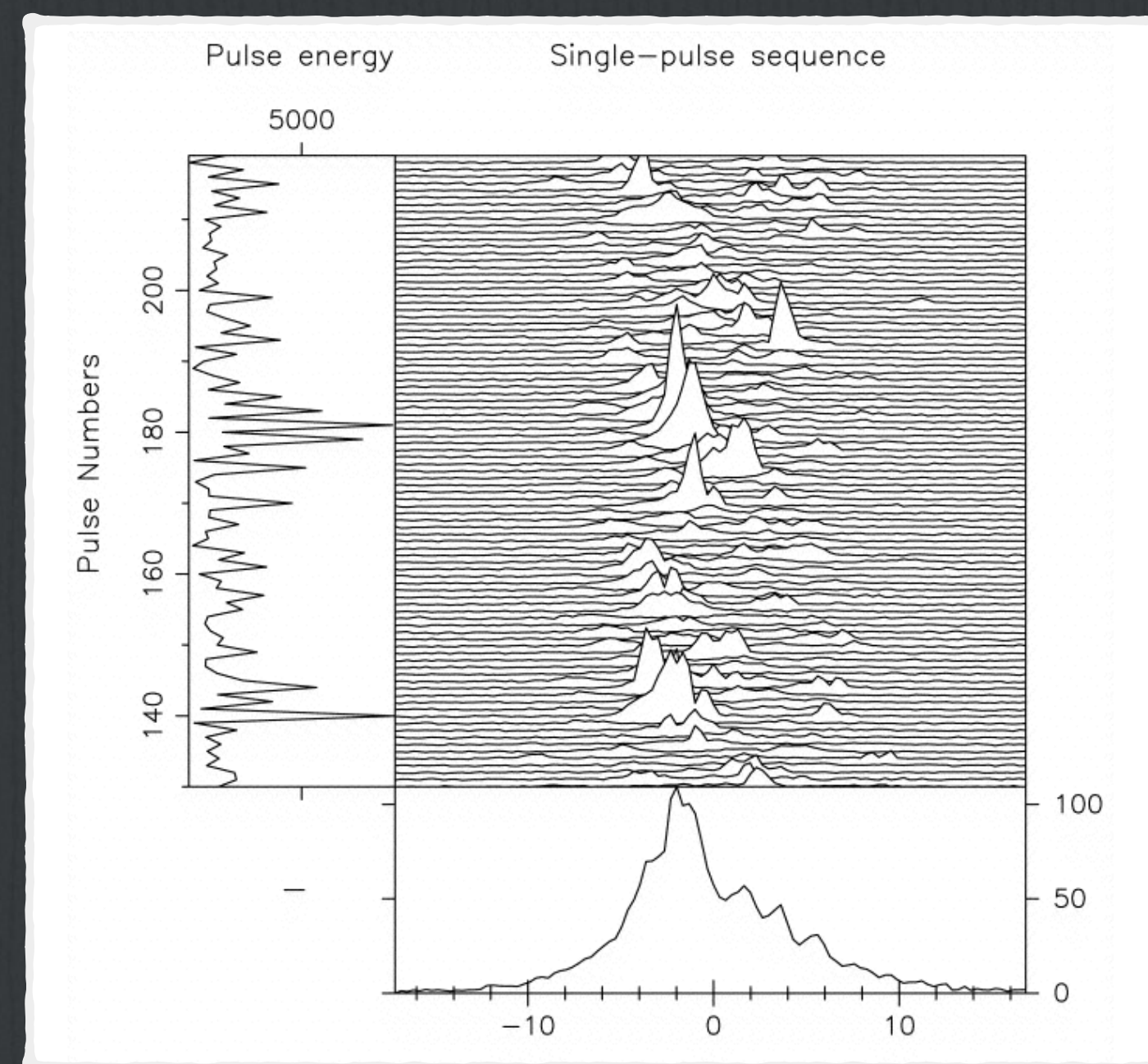
Topics we will cover

- Creating a stable pulse profile through folding
- Pulsars are broad band emitters
- Contamination by RFI
- Propagation of pulsar signals through the Interstellar Medium

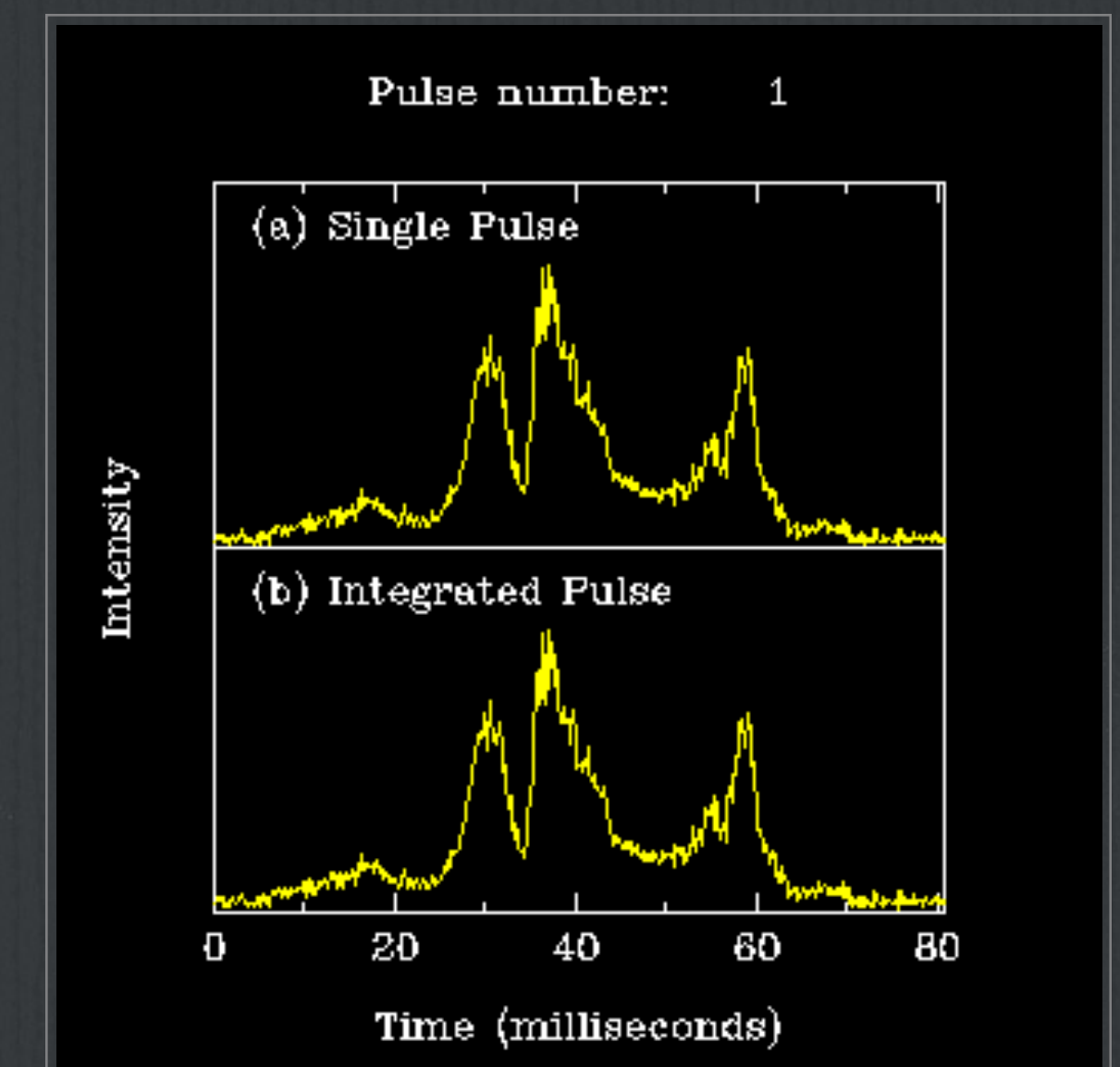
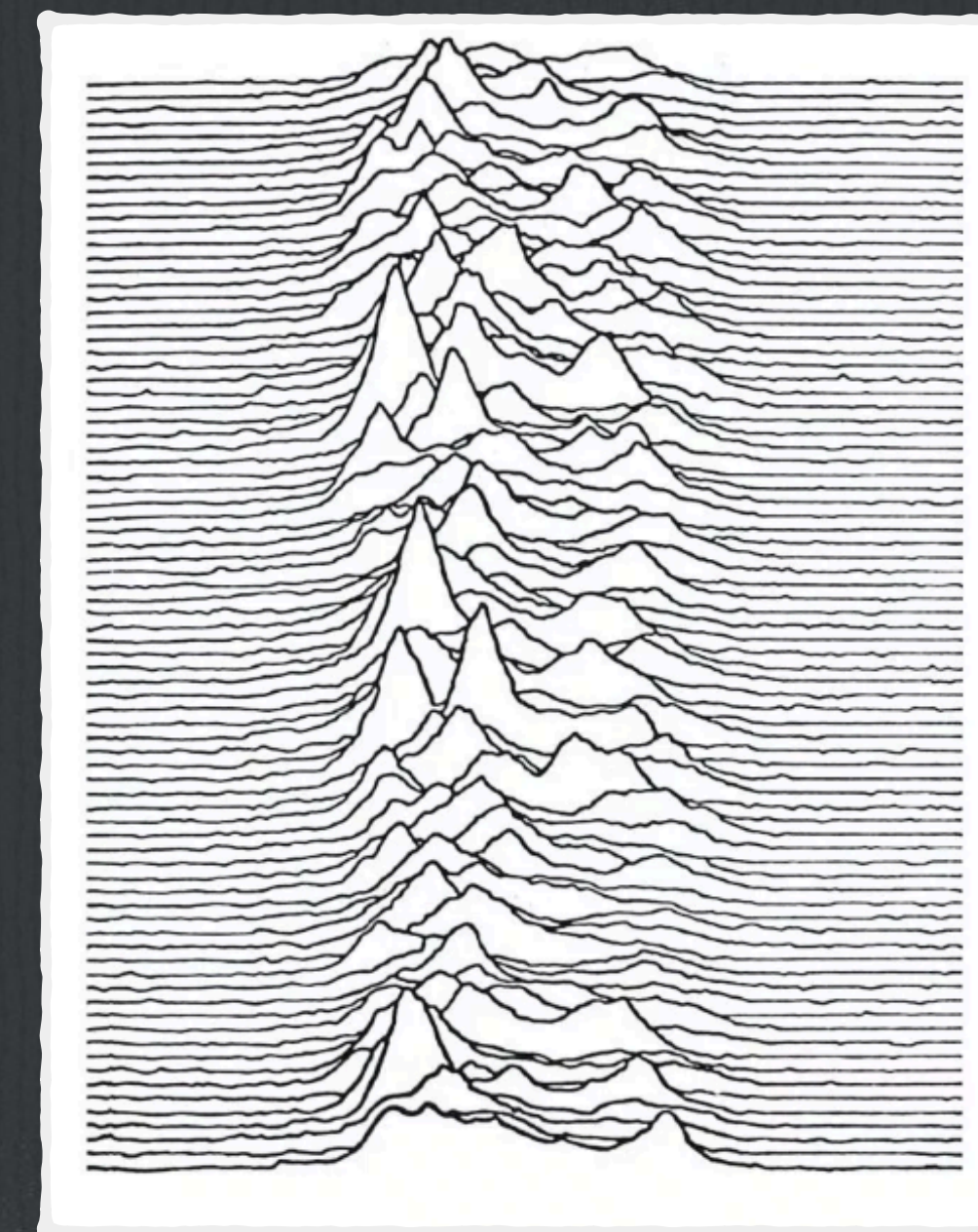
Folding pulsar signals to a stable pulse shape

Many pulsars are too weak to easily detect single pulses

- Moreover single pulses mostly appear varied in shape
- However, when we average together ~ 1000 pulses a higher S/N stable pulse shape is formed
- By knowing the pulsar observing parameters we can add single pulses together to form detectable average pulses
- This process of adding pulsar data to form a stable pulse shape is called 'folding'



PSR B0943+10



CP 1919 - the original pulsar

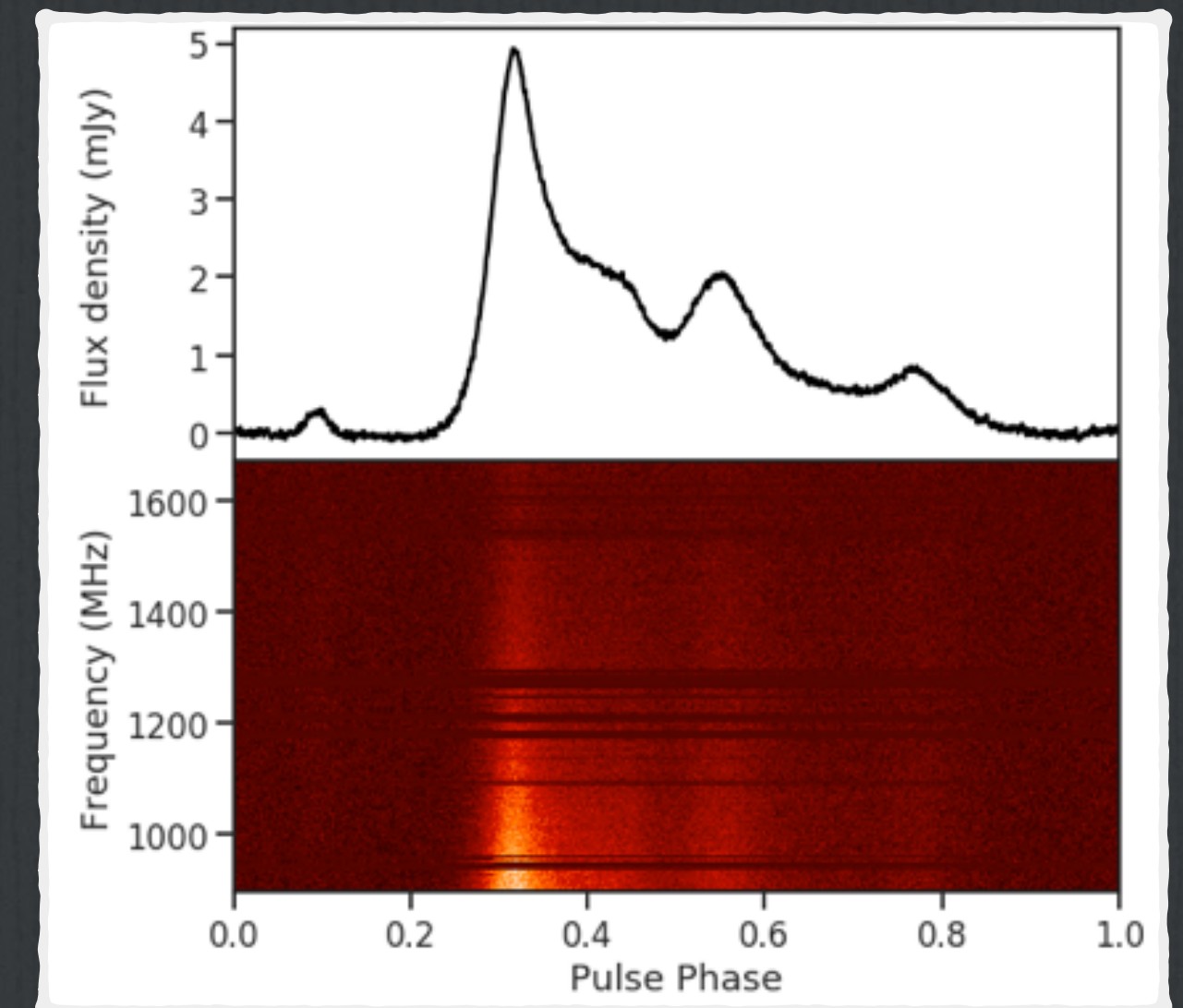
Pulsars are broadband radio emitters

- They have been observed in the radio all the way from 30 MHz - 230 GHz!
- They are intrinsically brighter at lower frequencies
- Which means they are steep spectrum sources:

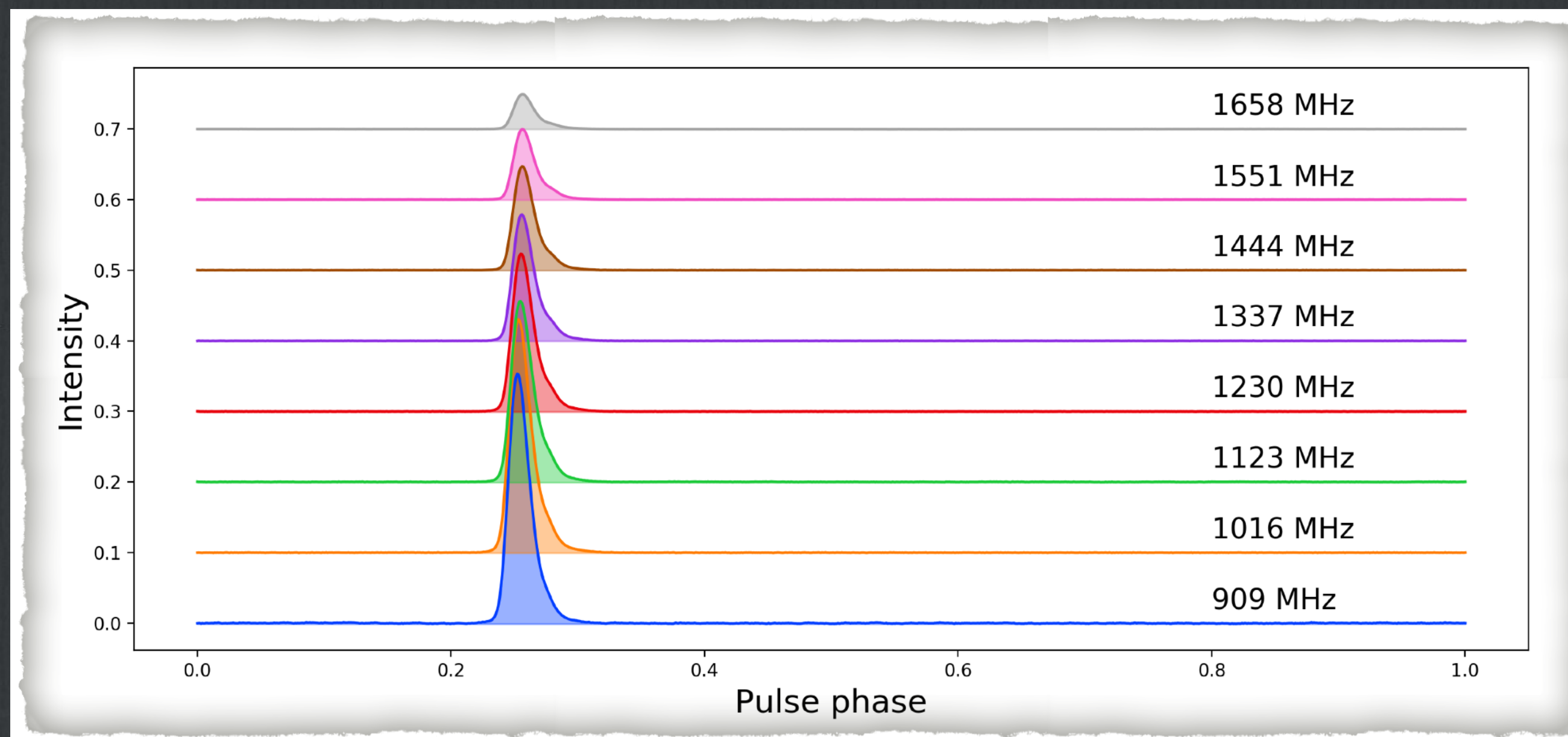
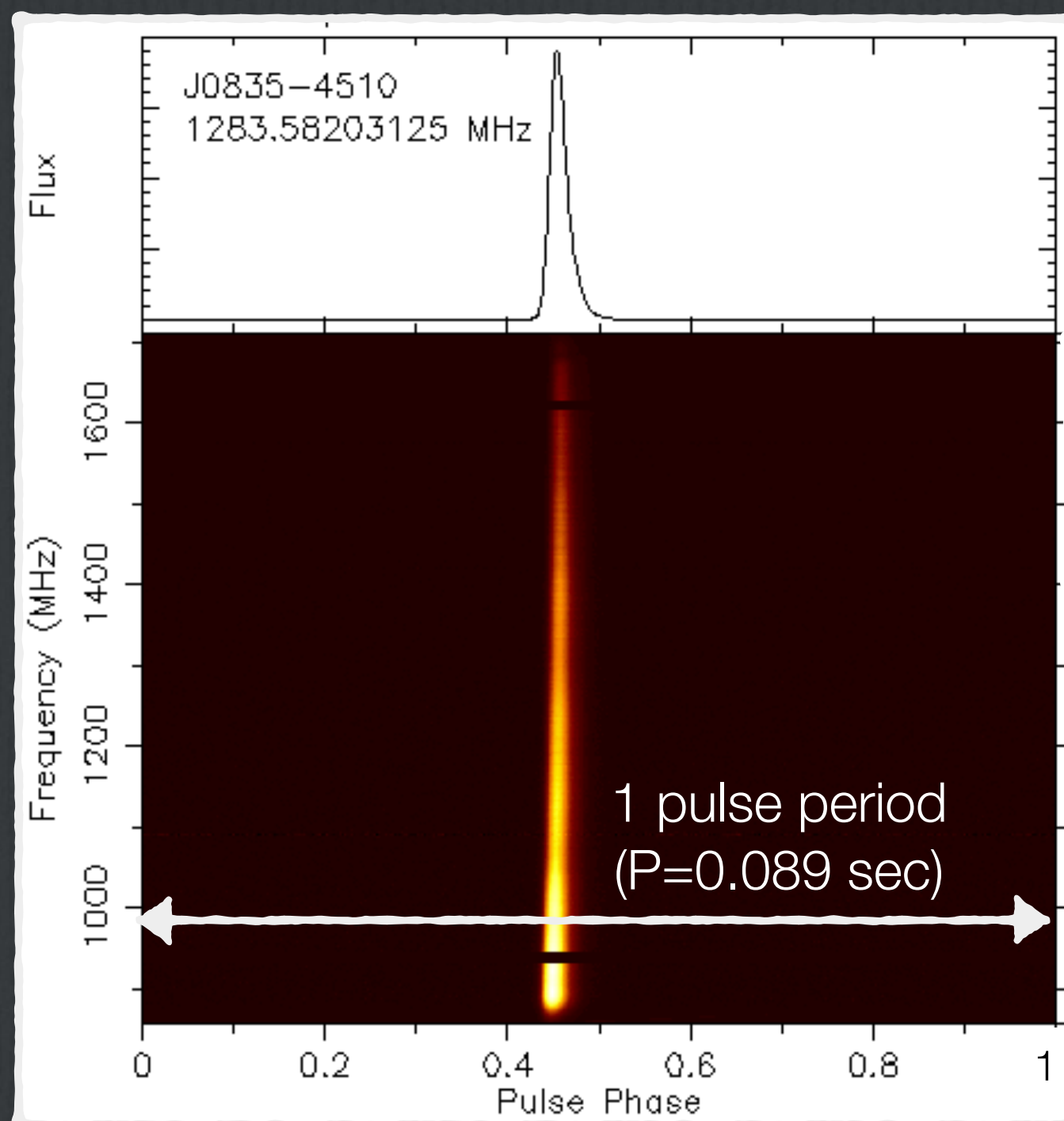
$$S_\nu \propto \nu^{-\alpha}$$

where S_ν is flux (Jy) at a frequency ν and typical values for $\alpha \sim 1.4$ to 1.8

PSR J0955-6150 across the MeerKAT L-band



The Vela pulsar (J0835-4510) at MeerKAT L-band

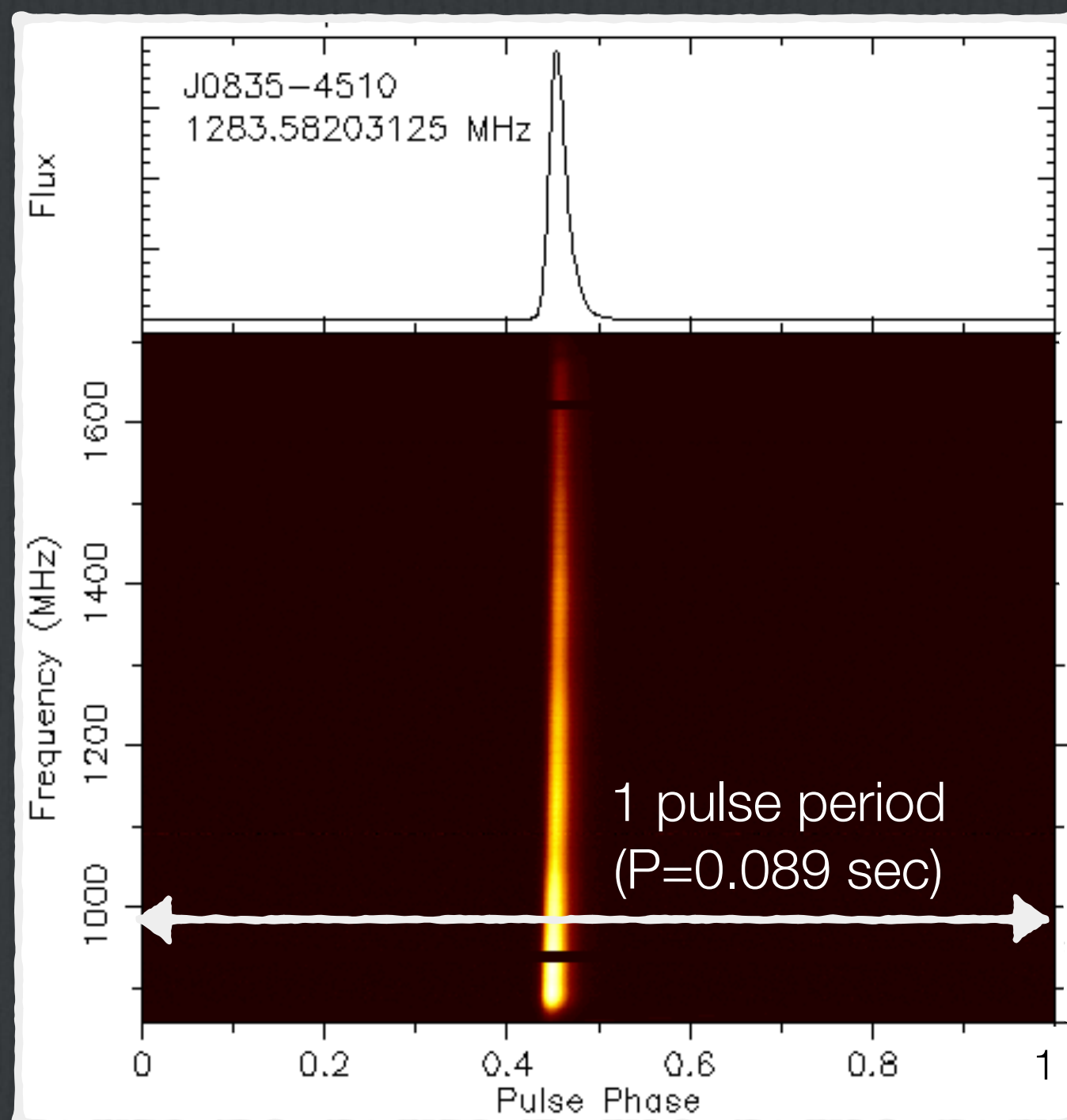
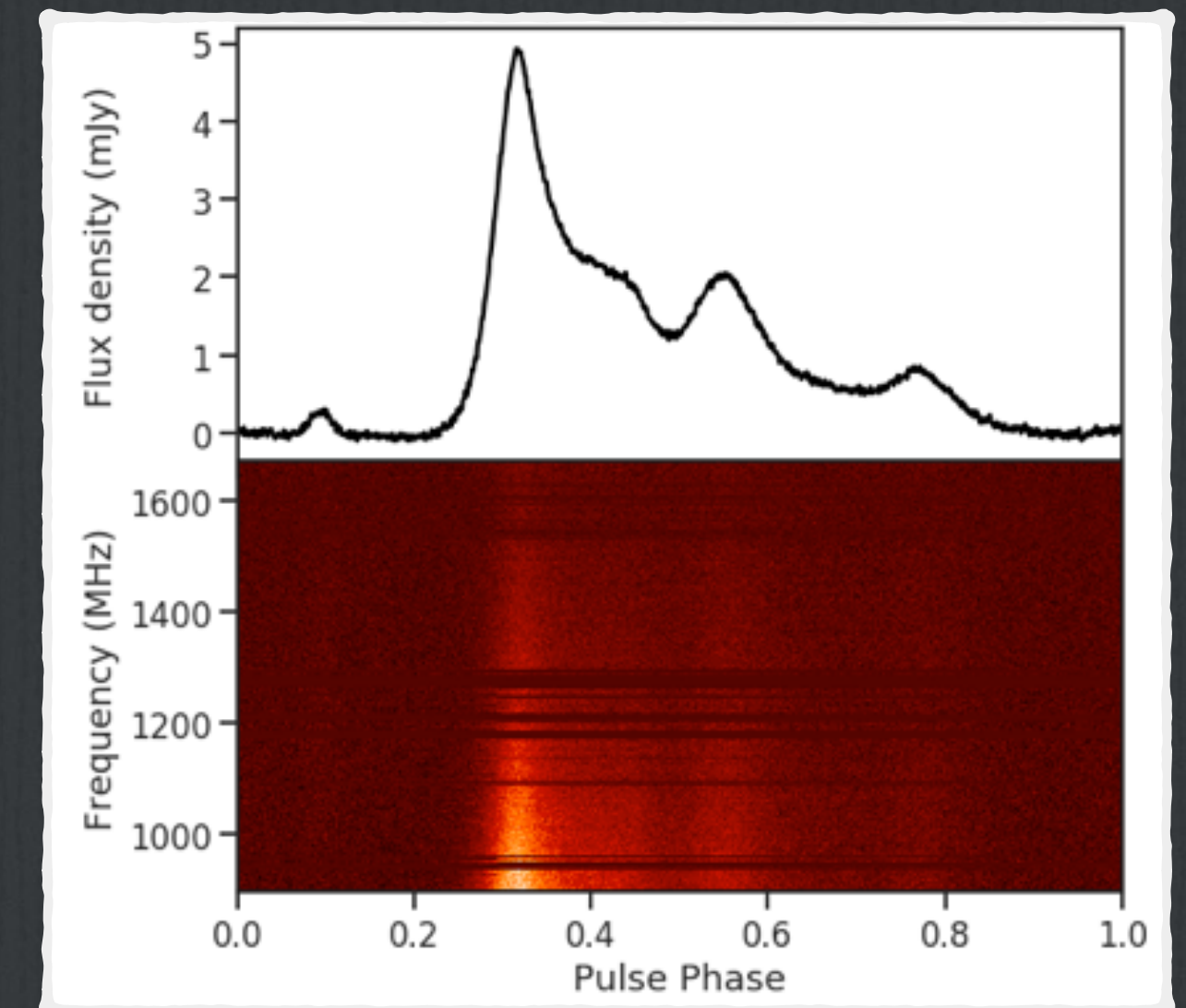


Pulsars are broadband radio emitters

- They have been observed in the radio all the way from 30 MHz - 230 GHz!
- They are intrinsically brighter at lower frequencies
- Which means they are steep spectrum sources:

$$S_\nu \propto \nu^{-\alpha}$$

where S_ν is flux (Jy) at a frequency ν and typical values for $\alpha \sim 1.4$ to 1.8



The observing frequency trade-off

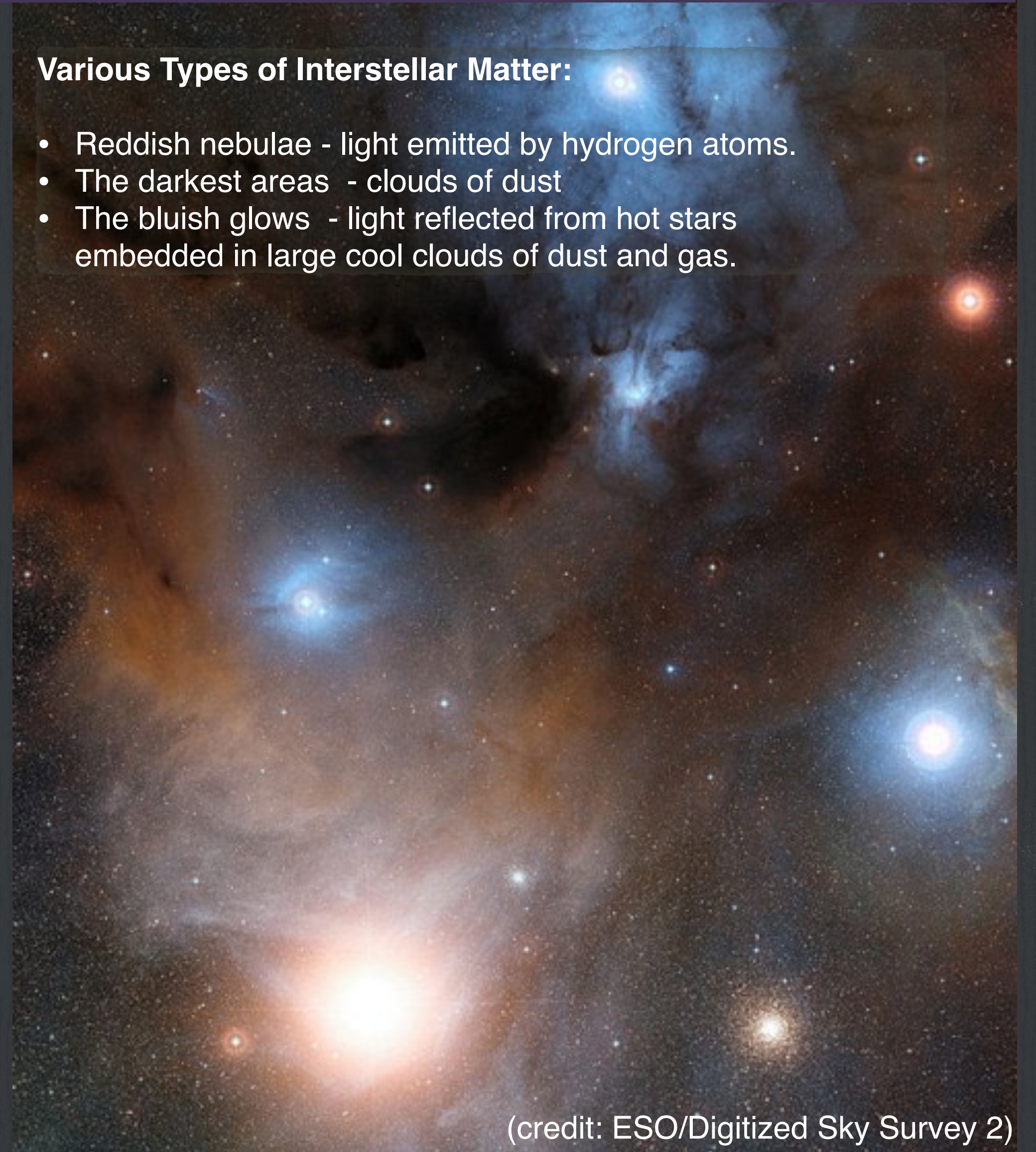
- Even though pulsars are brighter at lower frequencies, we'll see that lower frequency signals are more effected by the Interstellar Medium (ISM)
- Finding the ideal observing frequency is therefore a trade-off between intrinsic brightness and ISM effects

Propagation through the Interstellar Medium

- Here we investigate the effect of the Interstellar Medium in the propagation of radio signals - in general; and then consider the effects on pulsar signals in particular
- All stars and objects in the Galaxy are embedded in a diffuse medium, called the 'Interstellar Medium'
- The ISM has many different components, including
 - gas (containing atoms, molecules, ions and free electrons)
 - dust (tiny solid particles)

Various Types of Interstellar Matter:

- Reddish nebulae - light emitted by hydrogen atoms.
- The darkest areas - clouds of dust
- The bluish glows - light reflected from hot stars embedded in large cool clouds of dust and gas.



(credit: ESO/Digitized Sky Survey 2)

Propagation through the Interstellar Medium

- Here we investigate the effect of the Interstellar Medium in the propagation of radio signals - in general; and then consider the effects on pulsar signals in particular

- All stars and objects in the Galaxy are embedded in a diffuse medium, called the 'Interstellar Medium'

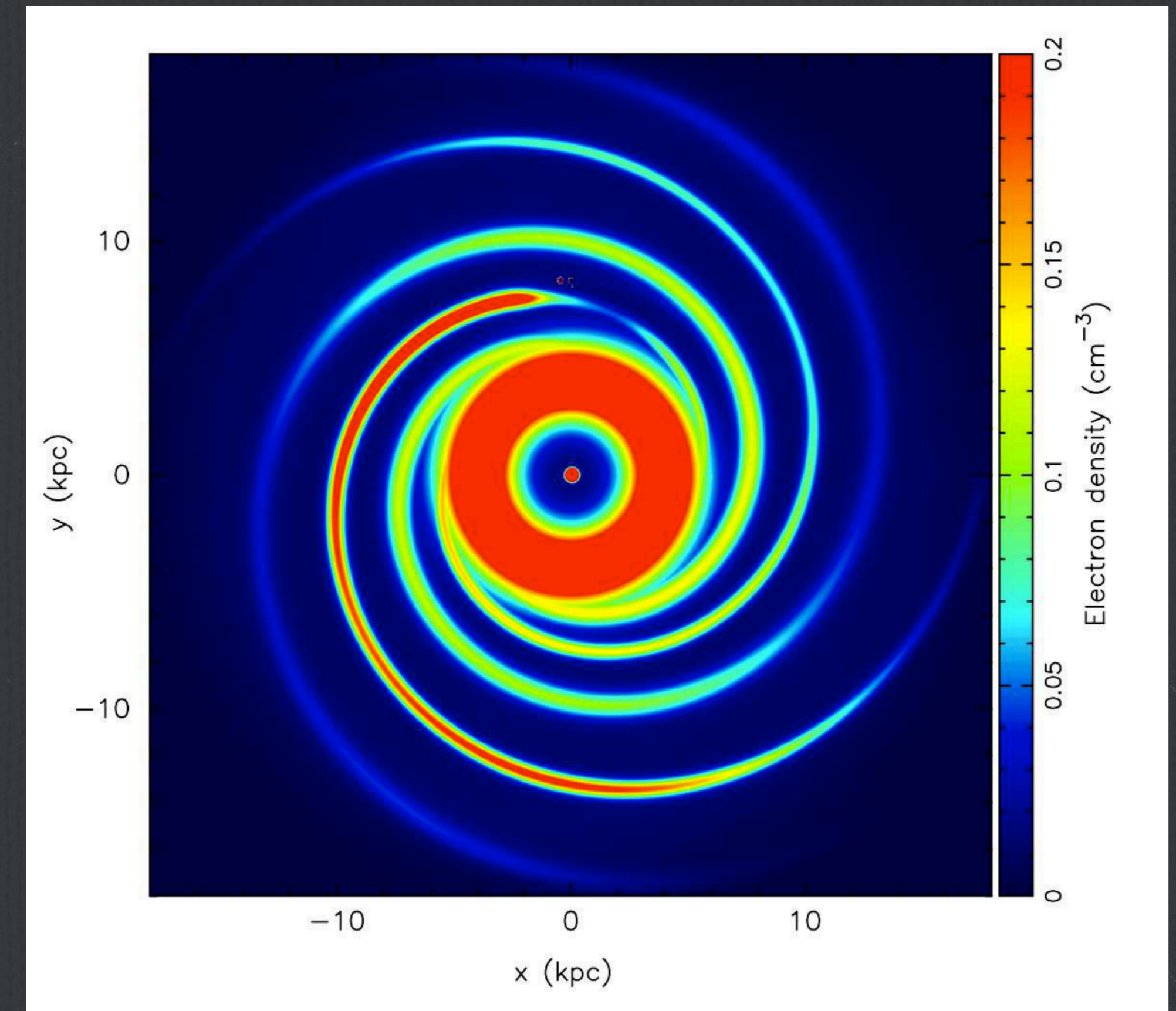
- The ISM has many different components, including

- gas (containing atoms, molecules, ions and free electrons)
- dust (tiny solid particles)

- This ionised ISM (containing free electrons) can be modelled as a cold** plasma of ~ 8000 K

- characterised by its line of sight electron densities

** note that cold means it is colder than the 'hot' or 'fully ionised' part of the ISM. In other words it is partially ionised.



Yao et al. 2017: Electron density model in the galactic plane

The ISM Cold Plasma refractive index

-The ISM refractive index (μ)

$$\mu = \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2}$$

- ν is the frequency of the radio emission propagating through the ISM
- ν_p is the resonant ISM plasma frequency as a function of free electron density (n_e)

$$\nu_p = \left(\frac{e^2 n_e}{\pi m_e}\right)^{1/2} \approx 8.97 \left(\frac{n_e}{\text{cm}^{-3}}\right)^{1/2} \text{ kHz}$$

electron mass, $m_e = 9.109 \times 10^{-31} \text{ kg}$
electron charge, $e = 1.6022 \times 10^{-19} \text{ C}$

The ISM Cold Plasma refractive index

-The ISM refractive index (μ)

$$\mu = \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2}$$

- ν is the frequency of the radio emission propagating through the ISM
- ν_p is the resonant ISM plasma frequency as a function of free electron density (n_e)

SI conversions
 $1 \text{ cm}^{-3} = 1 \times 10^{-6} \text{ m}^{-3}$
 $1 \text{ Hz} = 1000 \text{ kHz}$

typical average n_e densities in the galaxy $\sim 0.03 \text{ cm}^{-3}$

$$\nu_p = \left(\frac{e^2 n_e}{\pi m_e}\right)^{1/2} \approx 8.97 \left(\frac{n_e}{\text{cm}^{-3}}\right)^{1/2} \text{ kHz}$$

electron mass, $m_e = 9.109 \times 10^{-31} \text{ kg}$
electron charge, $e = 1.6022 \times 10^{-19} \text{ C}$

typical $\sim 1.5 \text{ kHz}$

Radio wave propagation through ISM

- Radio waves in the ISM propagate with group velocity v_g
- The ISM refractive index (μ) can be written as the ratio of group velocity and speed of light (c)

$$\mu = \frac{v_g}{c} = \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2}$$

$\nu_p > \nu$ then $\mu \rightarrow$ imaginary
waves don't propagate

$\nu_p < \nu$ then $\mu < 1$
waves propagate with velocity

$$v_g = c\mu < c$$

typical ~ 1.5 kHz

radio observations ~ 10 MHz - 100 GHz

Radio wave propagation through ISM

- Radio waves in the ISM propagate with group velocity v_g
- Since $\nu_p \ll \nu$ for radio observations we can compute v_g using a Taylor expansion

$$v_g = c \mu = c \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2} \approx c \left(1 - \frac{\nu_p^2}{2\nu^2}\right) \leftarrow \begin{array}{l} \text{first 2 terms} \\ \text{of Taylor expansion} \end{array}$$

Dispersive Delay by the ISM

$$\frac{1}{v_g} \approx \frac{1}{c} \left(1 + \frac{v_p^2}{2\nu^2} \right)$$

Radio emission of frequency ν is delayed by the ISM by t sec:

$$v_p = \left(\frac{e^2 n_e}{\pi m_e} \right)^{1/2}$$

$$t = \int_0^d \frac{dl}{v_g} - \frac{d}{c} = \frac{1}{c} \int_0^d \left(1 + \frac{v_p^2}{2\nu^2} \right) dl - \frac{d}{c} = \left(\frac{e^2}{2\pi m_e c} \right) \nu^{-2} \int_0^d n_e dl$$

Plugging in astronomical units

$$t = 4.149 \times 10^3 \text{sec} \left(\frac{\nu}{\text{MHz}} \right)^{-2} \left(\frac{\int_0^d n_e dl}{\text{pc cm}^{-3}} \right)$$

$$= 4.149 \times 10^3 \text{sec} \left(\frac{\nu}{\text{MHz}} \right)^{-2} \left(\frac{\text{DM}}{\text{pc cm}^{-3}} \right)$$

Astrophysical object at
distance d ,
Observer at 0

Time to travel from
object
to observer
at the speed of light
(no delay)

Dispersive Delay by the ISM

$$\frac{1}{v_g} \approx \frac{1}{c} \left(1 + \frac{v_p^2}{2\nu^2} \right)$$

Radio emission of frequency ν is delayed by the ISM by t sec:

$$v_p = \left(\frac{e^2 n_e}{\pi m_e} \right)^{1/2}$$

$$t = \int_0^d \frac{dl}{v_g} - \frac{d}{c} = \frac{1}{c} \int_0^d \left(1 + \frac{v_p^2}{2\nu^2} \right) dl - \frac{d}{c} = \left(\frac{e^2}{2\pi m_e c} \right) \nu^{-2} \int_0^d n_e dl$$

Plugging in astronomical units

$$t = 4.149 \times 10^3 \text{sec} \left(\frac{\nu}{\text{MHz}} \right)^{-2} \left(\frac{\int_0^d n_e dl}{\text{pc cm}^{-3}} \right)$$

Dispersion measure, in: pc cm⁻³

$$= 4.149 \times 10^3 \text{sec} \left(\frac{\nu}{\text{MHz}} \right)^{-2} \left(\frac{\text{DM}}{\text{pc cm}^{-3}} \right)$$

Astrophysical object at distance d ,
Observer at 0

Time to travel from object
to observer
at the speed of light
(no delay)

Dispersive Delay by the ISM

$$\frac{1}{v_g} \approx \frac{1}{c} \left(1 + \frac{v_p^2}{2\nu^2} \right)$$

$$v_p = \left(\frac{e^2 n_e}{\pi m_e} \right)^{1/2}$$

Radio emission of frequency ν is delayed by the ISM by t sec:

$$t = \int_0^d \frac{dl}{v_g} - \frac{d}{c} = \frac{1}{c} \int_0^d \left(1 + \frac{v_p^2}{2\nu^2} \right) dl - \frac{d}{c} = \left(\frac{e^2}{2\pi m_e c} \right) \nu^{-2} \int_0^d n_e dl$$

Plugging in astronomical units

$$t = 4.149 \times 10^3 \text{sec} \left(\frac{\nu}{\text{MHz}} \right)^{-2} \left(\frac{\int_0^d n_e dl}{\text{pc cm}^{-3}} \right)$$

Dispersion
measure,
in: pc cm⁻³

$$= 4.149 \times 10^3 \text{sec} \left(\frac{\nu}{\text{MHz}} \right)^{-2} \left(\frac{\text{DM}}{\text{pc cm}^{-3}} \right)$$

Strong frequency dependence

Astrophysical object at
distance d ,
Observer at 0

Time to travel from
object
to observer
at the speed of light
(no delay)

Delay between radio frequencies by the ISM

Delay at 1 GHz (compared to speed of light)

$$t = 4.149 \times 10^3 \text{sec} \left(\frac{\nu}{\text{MHz}} \right)^{-2} \left(\frac{\text{DM}}{\text{pc cm}^{-3}} \right)$$

$\nu = 1000$

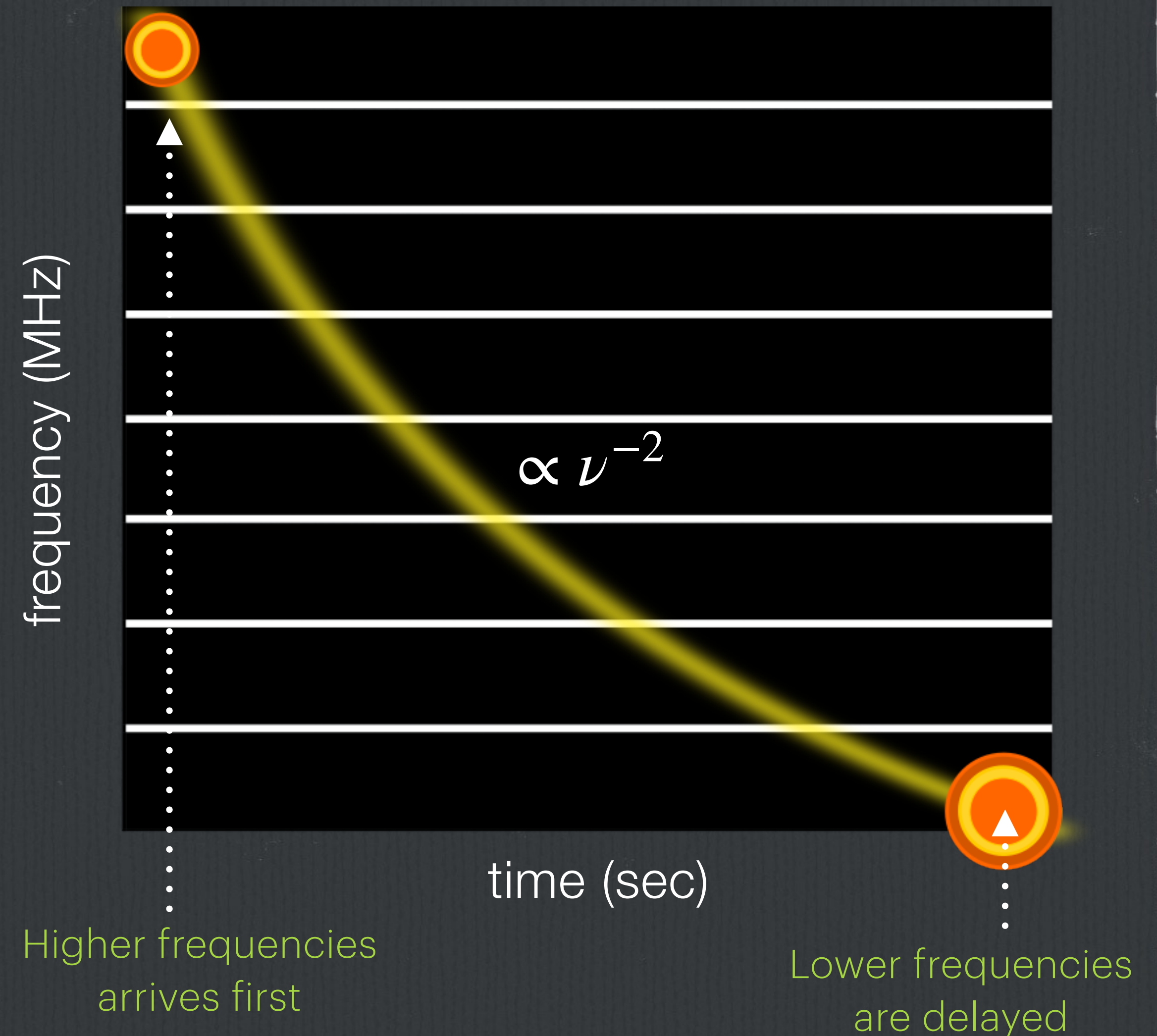
$$t = 4.149 \text{ms} \left(\frac{\text{DM}}{\text{pc cm}^{-3}} \right)$$

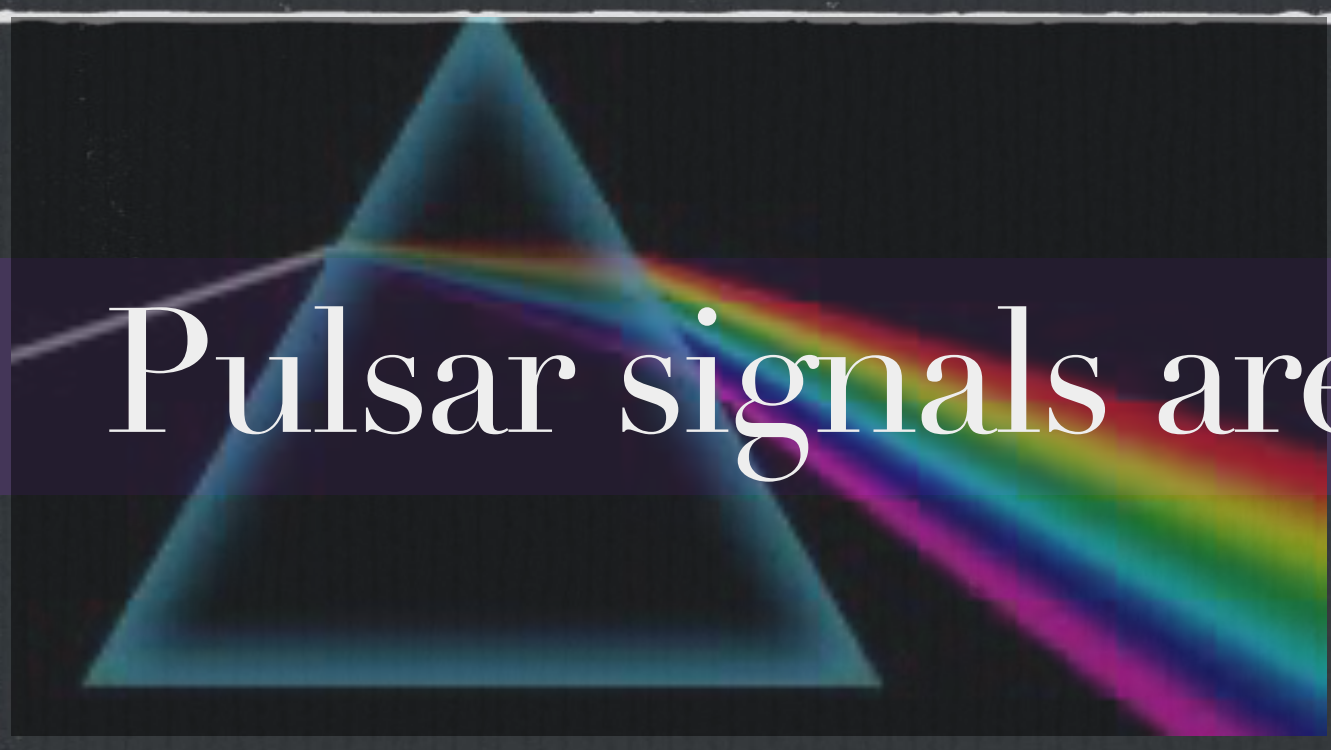
With DM typically 100 - 1000 pc cm⁻³,
the delay at 1 GHz is 100 ms to 1 sec

Delay between two frequencies

$$\Delta t = 4.15 \times 10^3 \text{sec} \times \text{DM} \times \left[\frac{1}{\nu_{\text{low}}^2} - \frac{1}{\nu_{\text{high}}^2} \right]$$

Higher frequencies arrives first





Pulsar signals are dispersed by the ISM

Radio waves pass through the interstellar medium (ISM) as they travel towards us

The electron densities in the partially ionised component of the ISM, acts as a frequency dependent dispersive plasma

This plasma will **slightly delay** a radio signal: at 1 GHz and at few 100 pc (~ 1000 light years), the delay is typically < 1 sec



Pulsar signals are dispersed by the ISM

Radio waves pass through the interstellar medium (ISM) as they travel towards us

The electron densities in the partially ionised component of the ISM, acts as a frequency dependent dispersive plasma

This plasma will **slightly delay** a radio signal: at 1 GHz and at few 100 pc (~ 1000 light years), the delay is typically < 1 sec

Most radio observations will be unaffected by this slight delay, but

1. Since pulsars are emitting signals every few ms to $\sim$ sec, this slight delay will affect the observed pulsar signal!
2. Moreover since pulsars are **broadband** — the ISM will *stretch out the pulsar signal*, such that



Pulsar signals are dispersed by the ISM

Radio waves pass through the interstellar medium (ISM) as they travel towards us

The electron densities in the partially ionised component of the ISM, acts as a frequency dependent dispersive plasma

This plasma will **slightly delay** a radio signal: at 1 GHz and at few 100 pc (~1000 light years), the delay is typically < 1 sec

Most radio observations will be unaffected by this slight delay, but

1. Since pulsars are emitting signals every few ms to ~ sec, this slight delay will affect the observed pulsar signal!
2. Moreover since pulsars are **broadband** — the ISM will *stretch out the pulsar signal*, such that

We observe the higher frequency components of the pulsar signal first, and the lower frequencies later

The time difference between two frequencies, ν_{high} and ν_{low} , is given by $\Delta t = 4.15 \times 10^3 \times \text{DM} \times \left[\frac{1}{\nu_{\text{low}}^2} - \frac{1}{\nu_{\text{high}}^2} \right]$

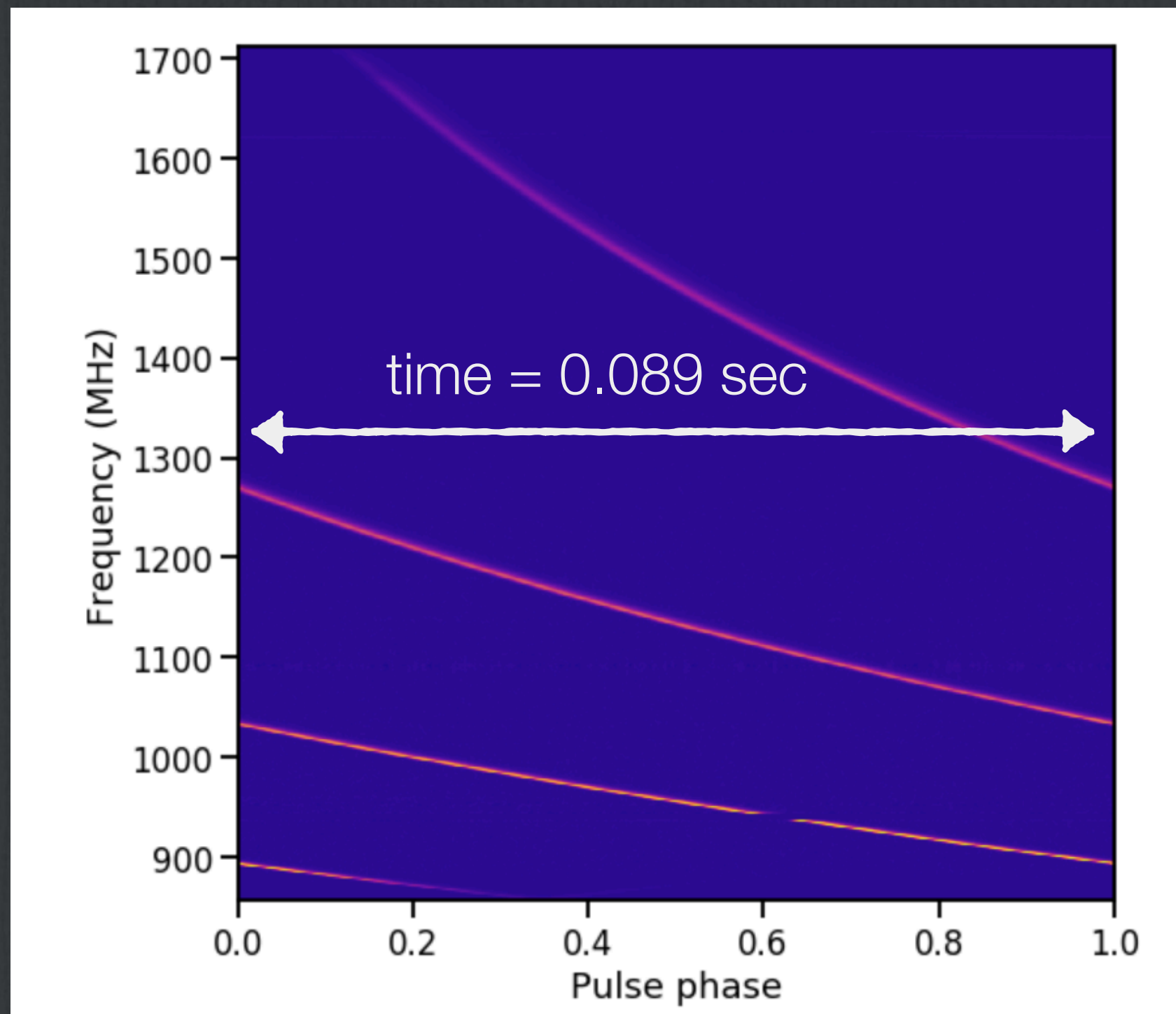
where the “dispersion measure” (DM) is the integrated electron column density, between the observer and the pulsar (in pc cm⁻³):

$$\text{DM} = \int_{\text{obs}}^{\text{pulsar}} n_e dl \quad \text{and therefore also a proxy for distance}$$

Pulsar signals are dispersed by the ISM

$$\Delta t = 4.15 \times 10^3 \times \text{DM} \times \left[\frac{1}{\nu_{\text{low}}^2} - \frac{1}{\nu_{\text{high}}^2} \right]$$

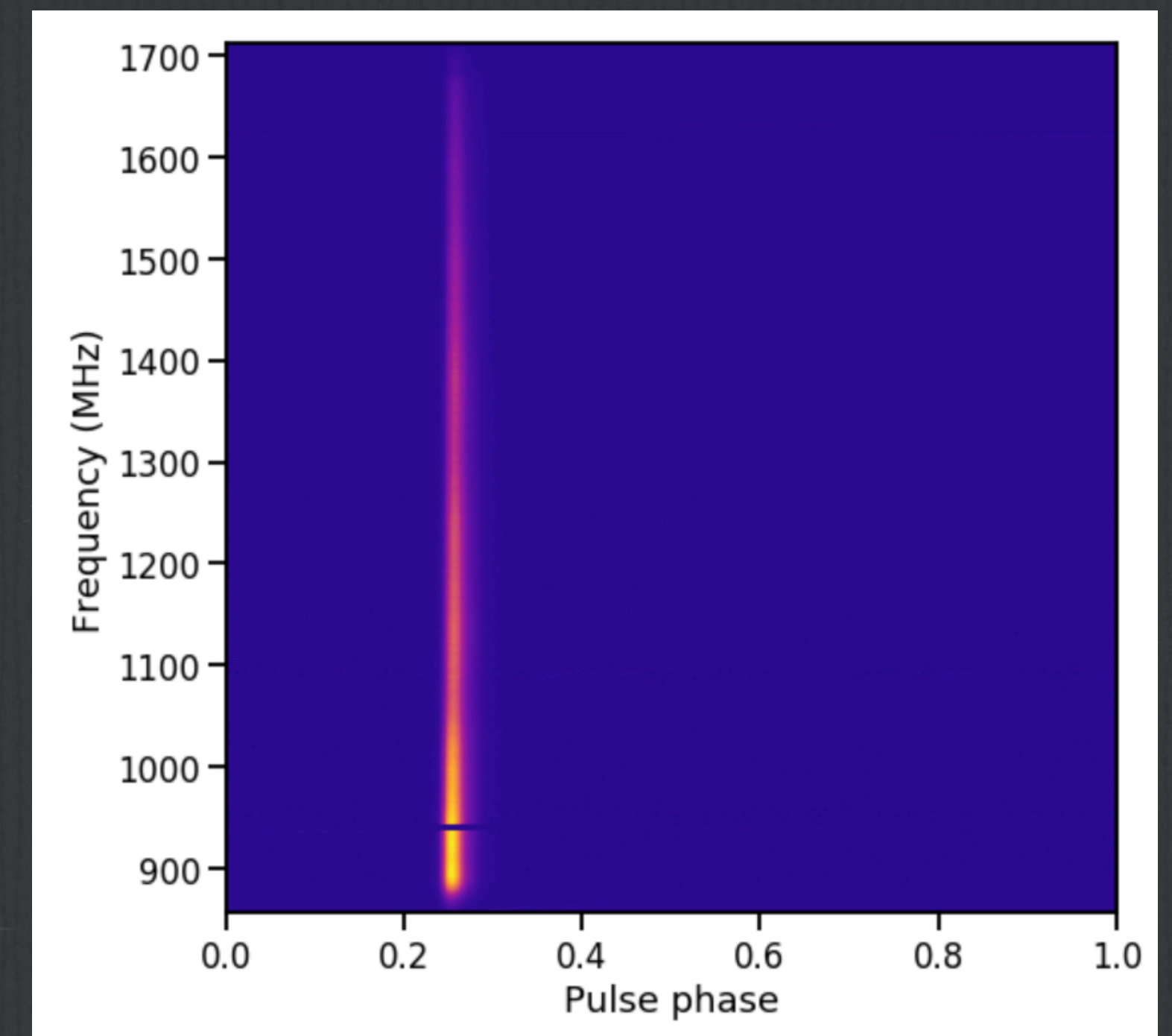
The Vela pulsar (J0835-4510) at MeerKAT L-band

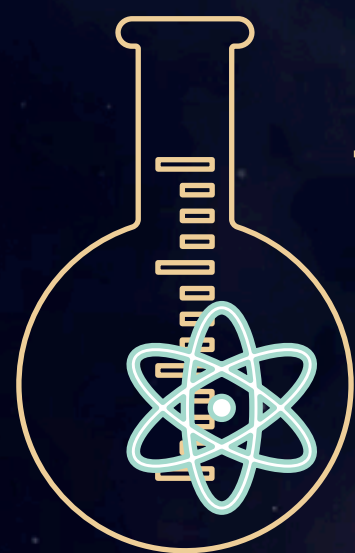


The Vela signal has been folded on its pulse period of 89 ms

However, the dispersive delay between the highest frequency and the lowest is larger than the pulse period

Vela after dispersion correction



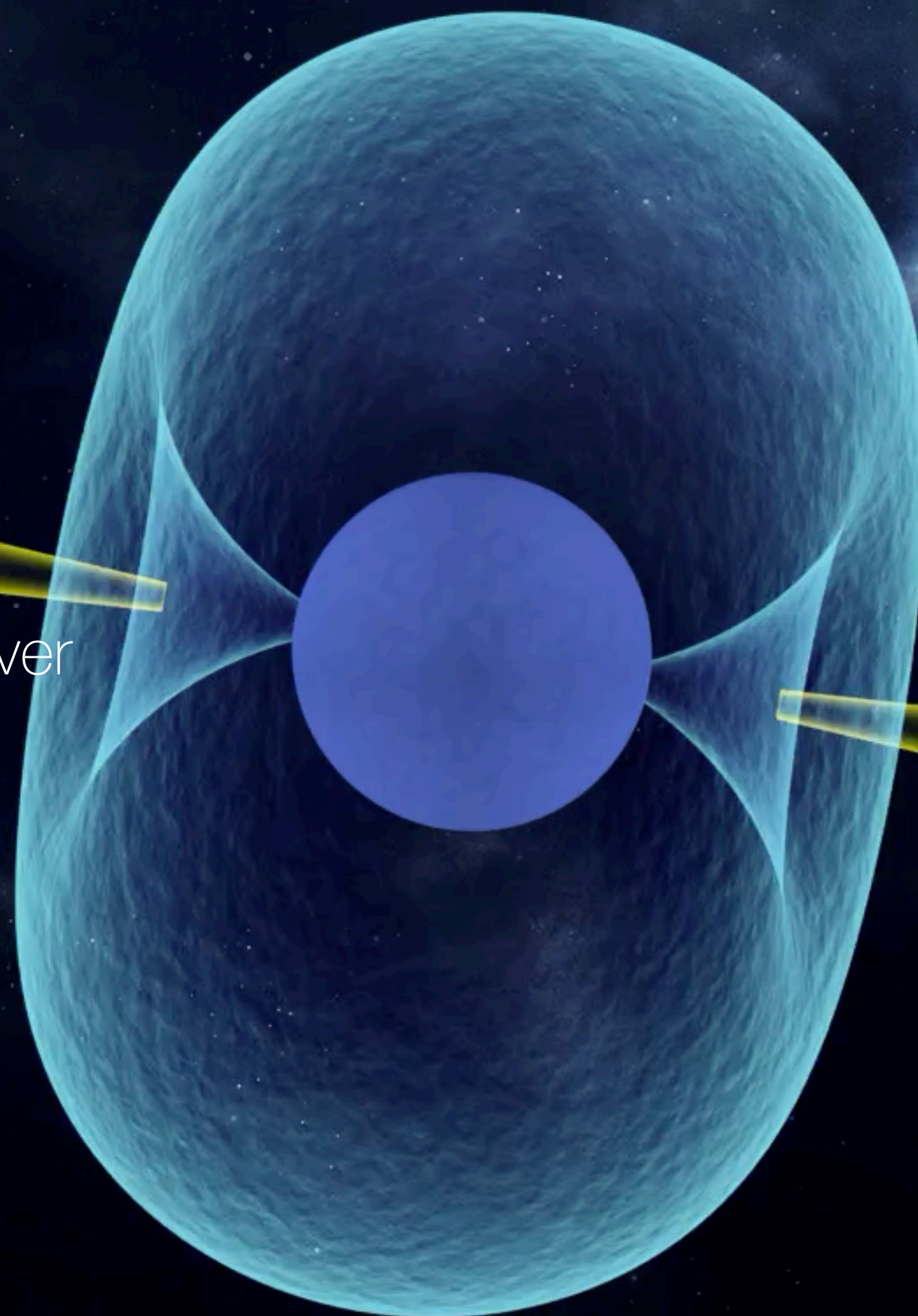


The extreme laboratory of pulsed neutron stars

emits coherent radio beams
along its magnetic axis

incredible magnetic fields ($10^{11} - 10^{14}$ G, over
a trillion times Earth's magnetic field)

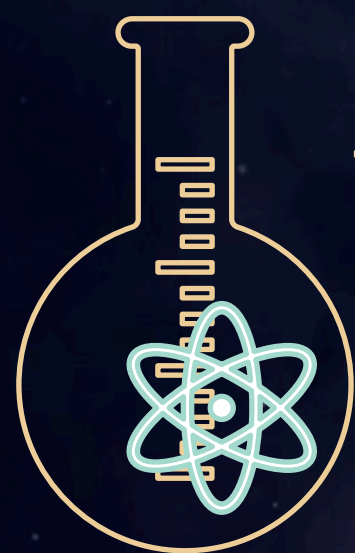
dead stellar core, spins rapidly
($P \sim 1.5$ ms - 2 sec), and with
incredible stability



remnant that remain is a neutron star,
radius ~ 10 km, mass of $\sim 1.4 M_{\text{sun}}$

ultra dense,
 $\sim 10\times$ nuclear densities

gravitational field
 2×10^{11} times Earth's

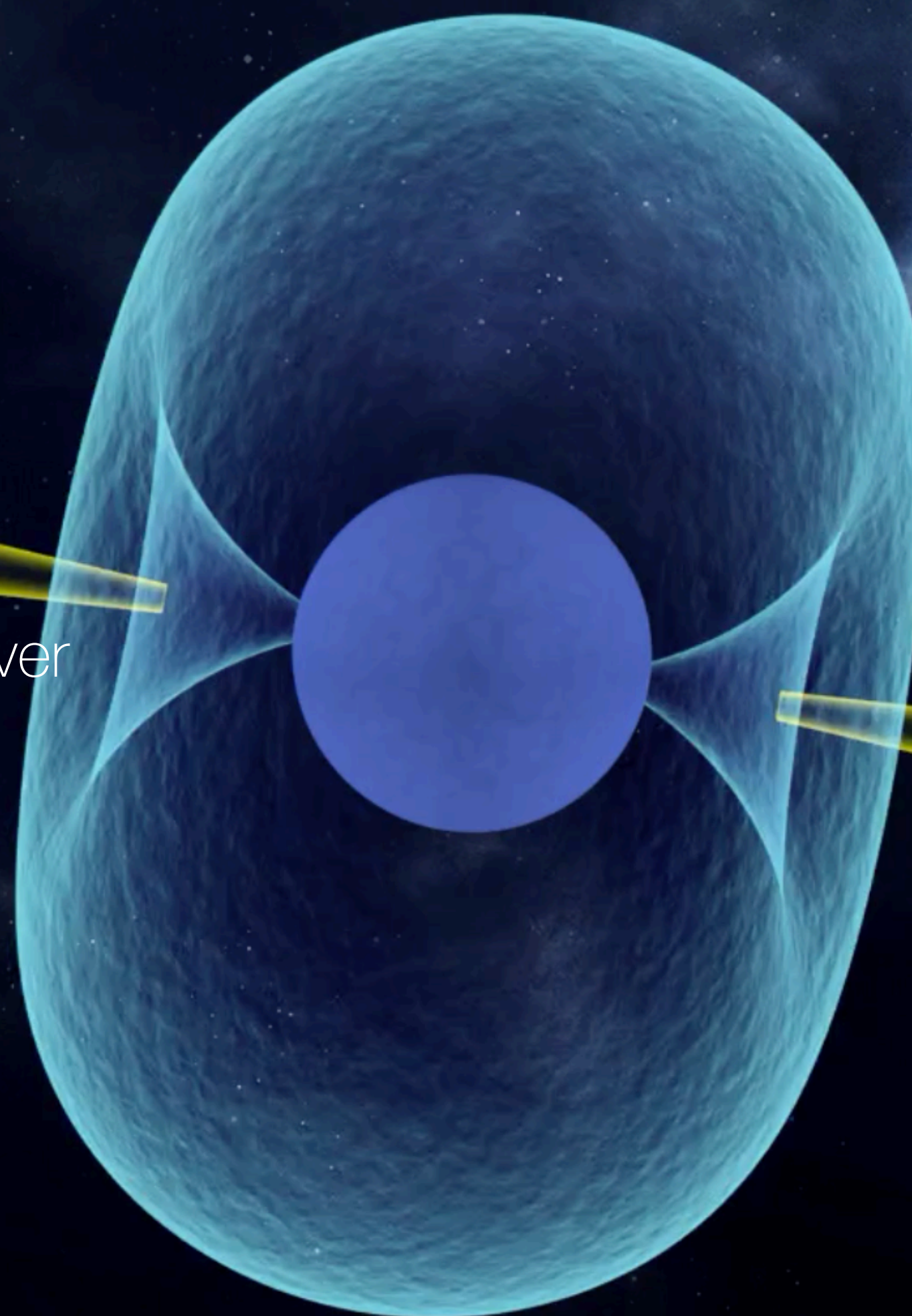


The extreme laboratory of pulsed neutron stars

emits coherent radio beams
along its magnetic axis

incredible magnetic fields ($10^{11} - 10^{14}$ G, over
a trillion times Earth's magnetic field)

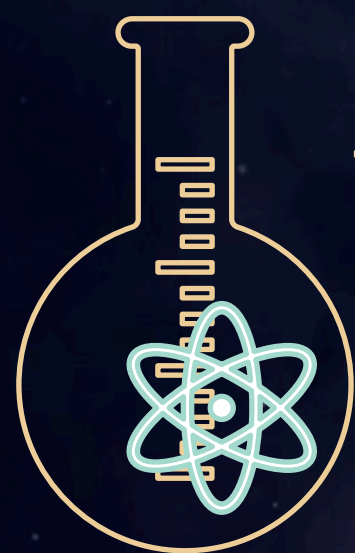
dead stellar core, spins rapidly
($P \sim 1.5$ ms - 2 sec), and with
incredible stability



 **point masses**

ultra dense,
 $\sim 10\times$ nuclear densities

gravitational field
 2×10^{11} times Earth's

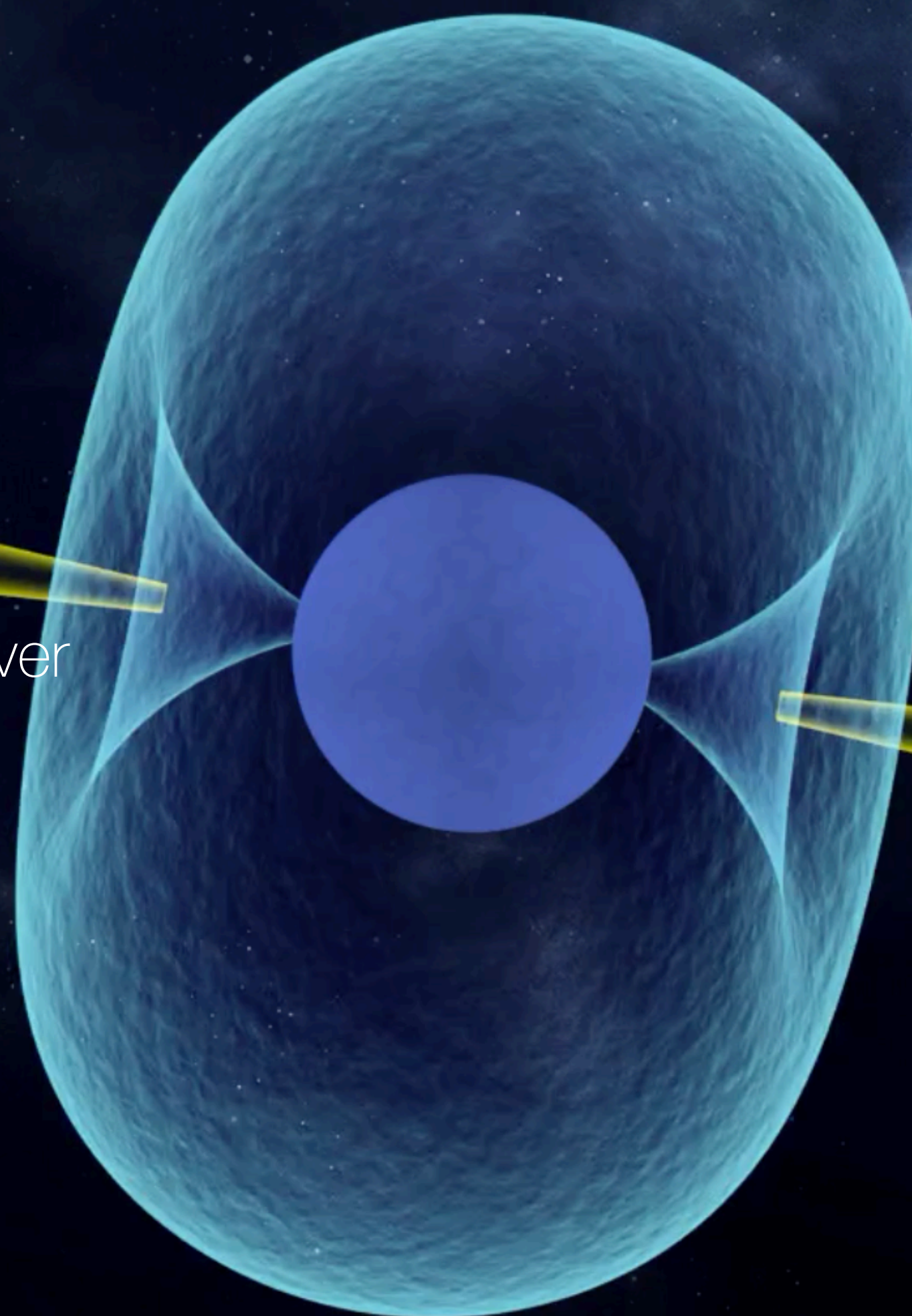


The extreme laboratory of pulsed neutron stars

emits coherent radio beams
along its magnetic axis

incredible magnetic fields ($10^{11} - 10^{14}$ G, over
a trillion times Earth's magnetic field)

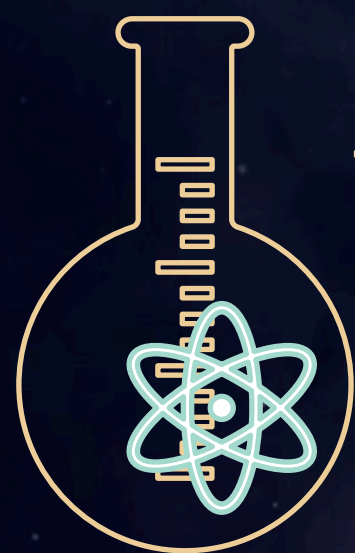
dead stellar core, spins rapidly
($P \sim 1.5$ ms - 2 sec), and with
incredible stability



 point masses

 exotic matter

gravitational field
 2×10^{11} times Earth's

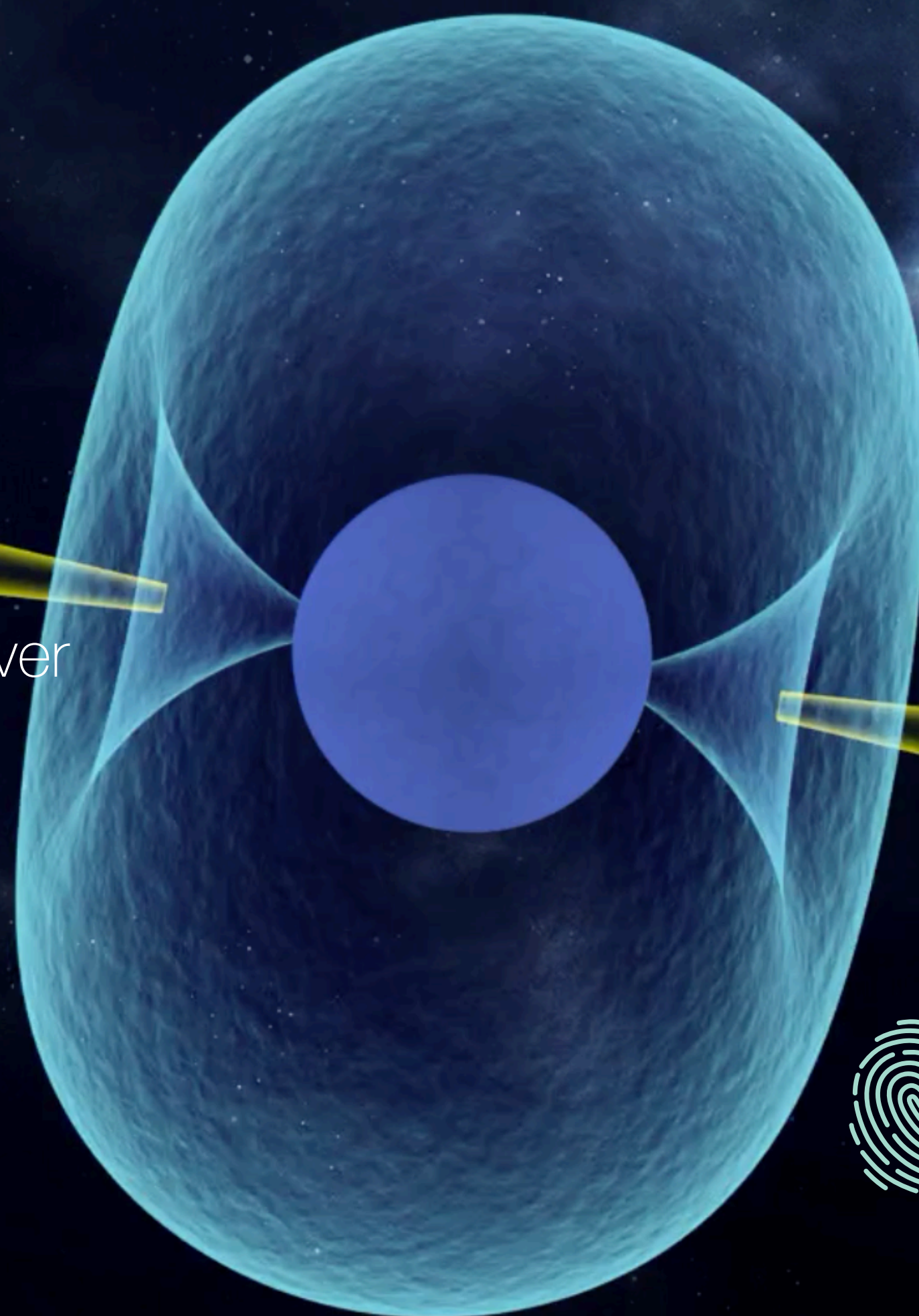


The extreme laboratory of pulsed neutron stars

emits coherent radio beams
along its magnetic axis

incredible magnetic fields ($10^{11} - 10^{14}$ G, over
a trillion times Earth's magnetic field)

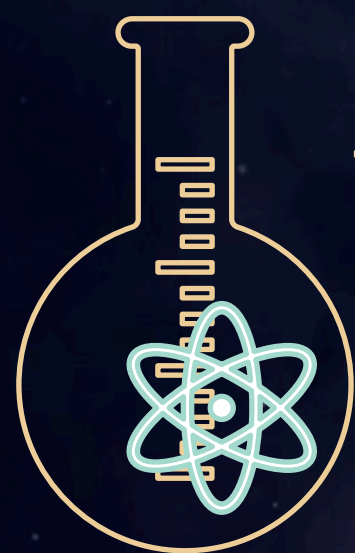
dead stellar core, spins rapidly
($P \sim 1.5$ ms - 2 sec), and with
incredible stability



 point masses

 exotic matter

 extreme gravity



The extreme laboratory of pulsed neutron stars

emits coherent radio beams
along its magnetic axis

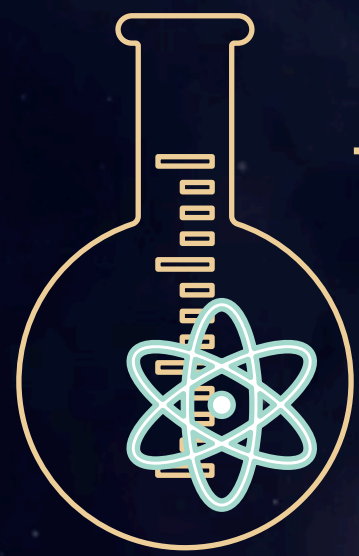
incredible magnetic fields (10^{11} - 10^{14} G, over
a trillion times Earth's magnetic field)

 accurate clocks

 point masses

 exotic matter

 extreme gravity



The extreme laboratory of pulsed neutron stars



interstellar medium
messengers



point masses



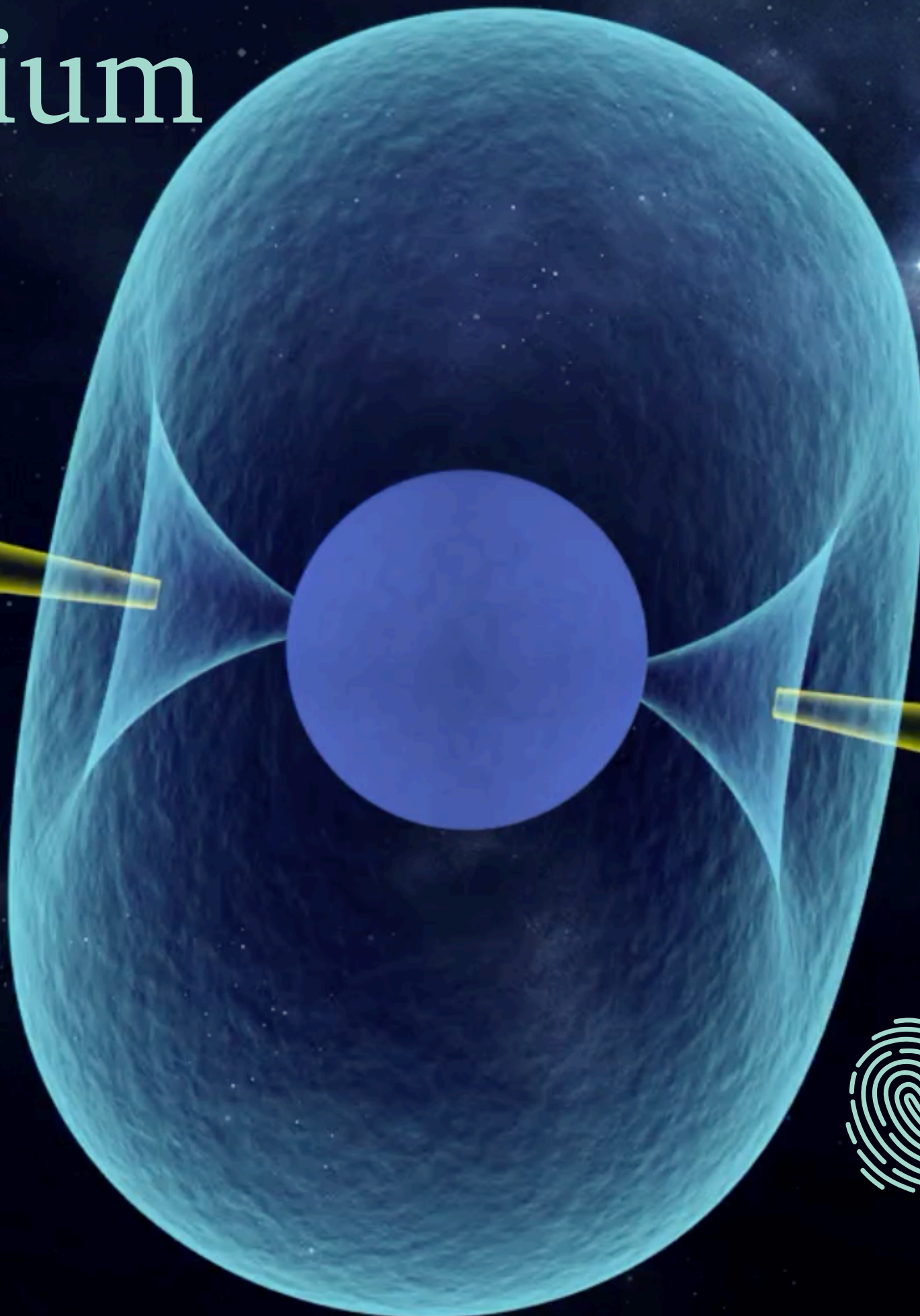
exotic matter

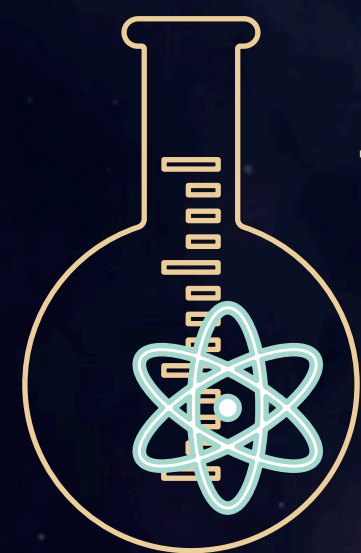


extreme gravity



accurate clocks





The extreme laboratory of pulsed neutron stars

interstellar medium
messengers

point masses

exotic matter

extreme gravity

Gravitational
Wave detectors

accurate clocks

Next up – Pulsar timing by Mayuresh



Extra Slides



Rotational energy

- Pulsars are observed to be spinning down with time —losing rotational energy
- We can estimate this rotational energy, and how it changes
- Rotational kinetic energy, E :

$$E = \frac{I\Omega^2}{2} = \frac{2\pi^2 I}{P^2}$$

Angular velocity
 $\Omega = 2\pi/P$

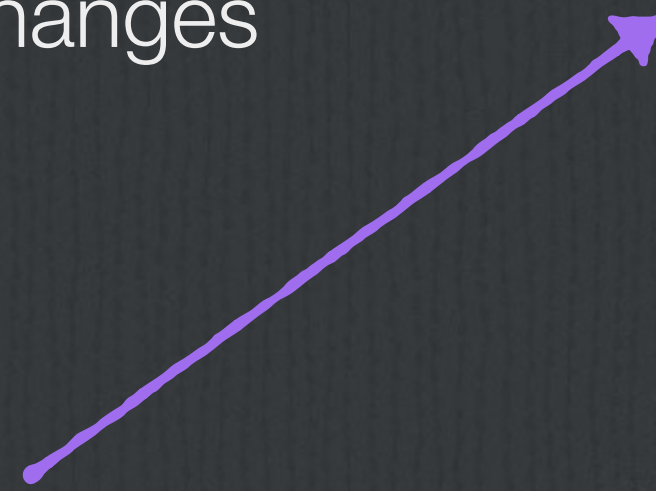


Artwork by Sandbox Studio, Chicago with Corinne Mucha

Rotational energy

- Pulsars are observed to be spinning down with time —losing rotational energy
- We can estimate this rotational energy, and how it changes
- Rotational kinetic energy, E :

$$E = \frac{I\Omega^2}{2} = \frac{2\pi^2 I}{P^2}$$



Canonical moment of inertia

$$\begin{aligned} I &= \frac{2MR^2}{5} \\ &= \frac{2 \times 1.4 \times 1.98 \times 10^{30} \text{ kg} \times (10^4 \text{ m})^2}{5} \\ &= 1.1 \times 10^{38} \text{ kg m}^2 \end{aligned}$$

Rotational energy

- Pulsars are observed to be spinning down with time —losing rotational energy
- We can estimate this rotational energy, and how it changes
- Rotational kinetic energy, E :

$$E = \frac{I\Omega^2}{2} = \frac{2\pi^2 I}{P^2}$$

Canonical moment of inertia

$$\begin{aligned} I &= \frac{2MR^2}{5} \\ &= \frac{2 \times 1.4 \times 1.98 \times 10^{30} \text{ kg} \times (10^4 \text{ m})^2}{5} \\ &= 1.1 \times 10^{38} \text{ kg m}^2 \end{aligned}$$

Pulse period (seconds)

Losing Rotational Energy

- From the observed changes in P we can estimate the loss in rotational energy.

From: $E = \frac{I\Omega^2}{2}$ 

$$\dot{E} = \frac{d}{dt} \left(\frac{1}{2} I \Omega^2 \right) = I \Omega \dot{\Omega}$$

$$\Omega = 2\pi/P$$

$$\dot{\Omega} = -\frac{2\pi\dot{P}}{P^2}$$

$$\dot{E} = -\frac{4\pi^2 I \dot{P}}{P^3}$$

P Pulse period (seconds)

$\dot{P}$ Change in pulse period
(seconds/seconds = unit-less)

Observables

Losing Rotational Energy

- From the observed changes in P we can estimate the loss in rotational energy.

From: $E = \frac{I\Omega^2}{2}$  $\dot{E} = \frac{d}{dt} \left(\frac{1}{2} I \Omega^2 \right) = I \Omega \dot{\Omega}$

$$\Omega = 2\pi/P$$

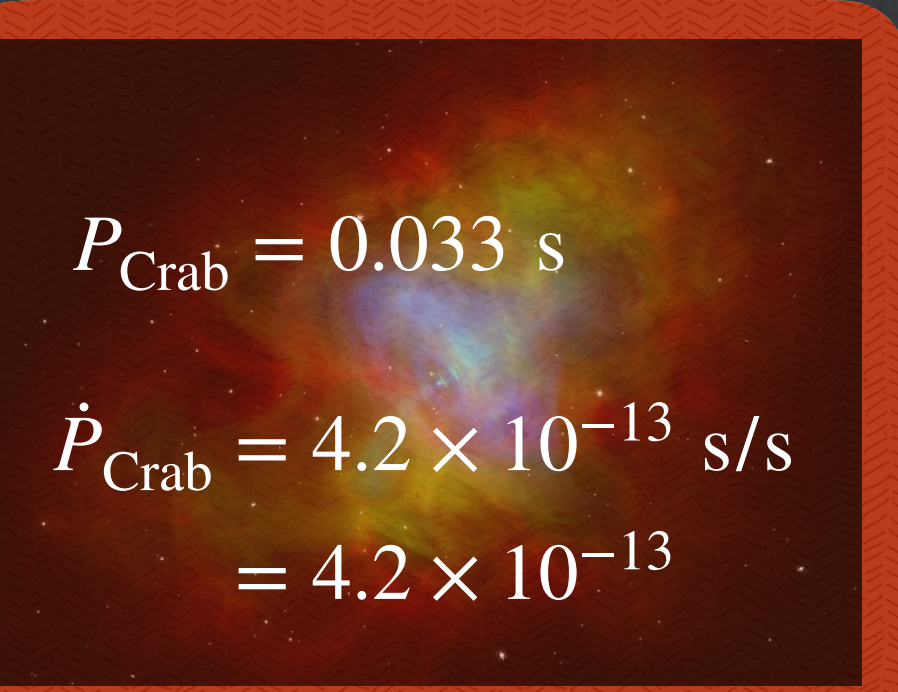
$$\dot{\Omega} = -\frac{2\pi\dot{P}}{P^2}$$

$$\dot{E} = -\frac{4\pi^2 I \dot{P}}{P^3}$$

P Pulse period (seconds)

$\dot{P}$ Change in pulse period
(seconds/seconds = unit-less)

Observables


$$P_{\text{Crab}} = 0.033 \text{ s}$$

$$\begin{aligned} \dot{P}_{\text{Crab}} &= 4.2 \times 10^{-13} \text{ s/s} \\ &= 4.2 \times 10^{-13} \end{aligned}$$

Losing Rotational Energy: Crab pulsar

$$\begin{aligned}\dot{E}_{\text{Crab}} &= -\frac{4\pi^2 I \dot{P}}{P^3} \\ &= \frac{4\pi^2 \times 1.1 \times 10^{38} \times 4.2 \times 10^{-13}}{(0.033)^3} W = 5 \times 10^{31} W \approx 10^5 L_{\odot}\end{aligned}$$

$$1 W = 1 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$$

Solar Luminosity

$$L_{\odot} = 3.83 \times 10^{26} W$$

The luminosity associated with this loss in rotational energy is comparable with the entire output of the Galaxy!

The dipolar radiation - emitted at the spin frequency ($1/P = \mathbf{30 \text{ Hz}}$) is absorbed by and heats up the surrounding Crab Nebula.



Composite image of the Crab Nebula.

Blue indicates X-rays (from Chandra), green is optical (from the HST), and red is radio (from the VLA). Image credit: J. Hester (ASU), CXC, HST, NRAO, NSF, NASA.

Estimating Magnetic Field

- Assuming the loss rotational energy is entirely due to the magnetic dipole radiation, we can estimate the magnetic field of a pulsar

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} \lesssim \frac{32 B^2 R^6 \pi^4}{3c^3 P^4} \quad \Rightarrow \quad B^2 \gtrsim \frac{3 I c^3}{8 \pi^2 R^6} \times P \dot{P}$$

Q: What is the minimum magnetic field of the Crab pulsar?

Estimating Magnetic Field

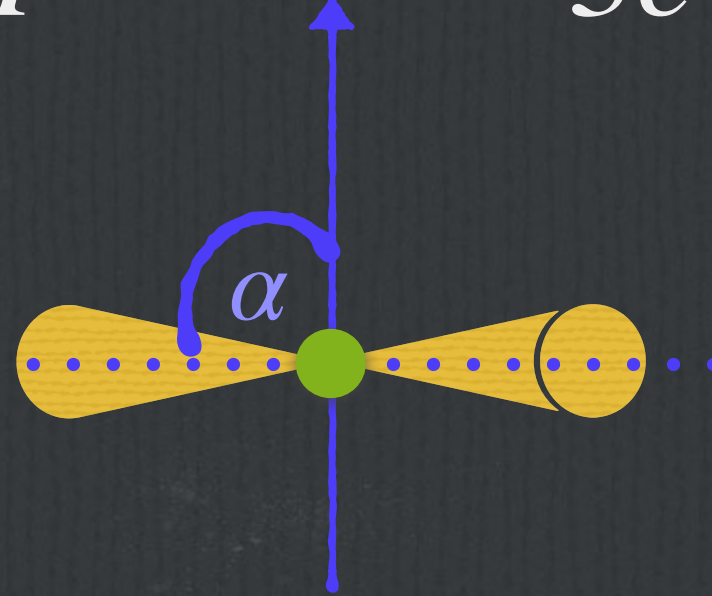
- Assuming the loss rotational energy is entirely due to the magnetic dipole radiation, we can estimate the magnetic field of a pulsar

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} \lesssim \frac{32 B^2 R^6 \pi^4}{3c^3 P^4}$$



$$B^2 \gtrsim \frac{3 I c^3}{8 \pi^2 R^6} \times P \dot{P}$$



‘Characteristic Age’

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} = \frac{32 B^2 R^6 \sin^2 \alpha \pi^4}{3c^3 P^4}$$

'Characteristic Age'

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{\cancel{P^3}} = \frac{32 B^2 R^6 \sin^2 \alpha \pi^4}{3c^3 \cancel{P^4}}$$



$$\dot{P} = KP^{-1}$$

First order differential Eq.

$$K = \frac{8 B^2 R^6 \sin^2 \alpha \pi^2}{3Ic^3}$$

'Characteristic Age'

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{\cancel{P^3}} = \frac{32 B^2 R^6 \sin^2 \alpha \pi^4}{3c^3 P^4}$$

First order differential Eq.



$$\dot{P} = KP^{-1}$$

$$\frac{dP}{dt} = \frac{K}{P}$$

$$\int_{P_0}^P P dP = K \int_0^{\tau_c} dt = \dot{P} P \int_0^{\tau_c} dt$$

$$K = \frac{8 B^2 R^6 \sin^2 \alpha \pi^2}{3Ic^3}$$

Notice K is assumed constant, so that

$$\dot{P}P = K = \text{const}$$

'Characteristic Age'

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} = \frac{32 B^2 R^6 \sin^2 \alpha \pi^4}{3c^3 P^4}$$

First order differential Eq.

$$\dot{P} = KP^{-1}$$

$$\frac{dP}{dt} = \frac{K}{P}$$

$$\int_{P_0}^P P dP = K \int_0^{\tau_c} dt = \dot{P} P \int_0^{\tau_c} dt$$

$$K = \frac{8 B^2 R^6 \sin^2 \alpha \pi^2}{3Ic^3}$$

Notice K is assumed constant, so that

$$\dot{P}P = K = \text{const}$$

Solving the integrals

$$\frac{1}{2} [P^2 - P_0^2] = \dot{P}P \tau_c$$

'Characteristic Age'

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} = \frac{32 B^2 R^6 \sin^2 \alpha \pi^4}{3c^3 P^4}$$

First order differential Eq.

$$\dot{P} = KP^{-1}$$

$$K = \frac{8 B^2 R^6 \sin^2 \alpha \pi^2}{3Ic^3}$$

Notice K is assumed constant, so that

$$\frac{dP}{dt} = \frac{K}{P}$$

$$\int_{P_0}^P P dP = K \int_0^{\tau_c} dt = \dot{P} P \int_0^{\tau_c} dt$$

Solving the integrals

Rewriting i.t.o τ_c the
"characteristic age"

$$\frac{1}{2} [P^2 - P_0^2] = \dot{P} P \tau_c$$

$$\tau_c = \frac{P}{2\dot{P}} \left[1 - \left(\frac{P_0}{P} \right)^2 \right] \approx \frac{P}{2\dot{P}}$$

$$P_0 \ll P$$

Birth period P_0
much less than
current P

'Characteristic Age'

An assumption!

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} = \frac{32 B^2 R^6 \sin^2 \alpha \pi^4}{3c^3 P^4}$$

First order differential Eq.

$$\dot{P} = KP^{-1}$$

$$K = \frac{8 B^2 R^6 \sin^2 \alpha \pi^2}{3Ic^3}$$

Notice K is assumed constant, so that

$$\frac{dP}{dt} = \frac{K}{P}$$

$$\int_{P_0}^P P dP = K \int_0^{\tau_c} dt = \dot{P} P \int_0^{\tau_c} dt$$

Solving the integrals

Rewriting i.t.o τ_c the
"characteristic age"

$$\frac{1}{2} [P^2 - P_0^2] = \dot{P} P \tau_c$$

$$\tau_c = \frac{P}{2\dot{P}} \left[1 - \left(\frac{P_0}{P} \right)^2 \right] \approx \frac{P}{2\dot{P}}$$

$$P_0 \ll P$$

Birth period P_0
much less than
current P

'Characteristic Age'

$$-\dot{E} = P_{\text{rad}}$$

$$\frac{4\pi^2 I \dot{P}}{P^3} = \frac{32 B^2 R^6 \sin^2 \alpha \pi^4}{3c^3 P^4}$$

First order differential Eq.

$$\dot{P} = KP^{-1}$$

$$K = \frac{8 B^2 R^6 \sin^2 \alpha \pi^2}{3Ic^3}$$

Notice K is assumed constant, so that

$$\frac{dP}{dt} = \frac{K}{P}$$

$$\int_{P_0}^P P dP = K \int_0^{\tau_c} dt = \dot{P} P \int_0^{\tau_c} dt$$

$$\frac{1}{2} [P^2 - P_0^2] = \dot{P} P \tau_c$$

$$\tau_c = \frac{P}{2\dot{P}} \left[1 - \left(\frac{P_0}{P} \right)^2 \right] \approx \frac{P}{2\dot{P}}$$

$$P_0 \ll P$$

Birth period P_0 much less than current P

Test it for the Crab pulsar, where we know the correct age of the pulsar through the SN age, which was observed in the year 1054.

Characteristic Age

$$\frac{P}{2\dot{P}} = 3.93 \times 10^{10} \text{s} \sim 1250 \text{ yr}$$

True Age

$$(2026 - 1054) \text{yr} = 972 \text{yr}$$

Solving the integrals

Rewriting i.t.o τ_c the "characteristic age"