Continuous Waves & Burst Signals in PTAs

Boris Goncharov

boris.goncharov@aei.mpg.de

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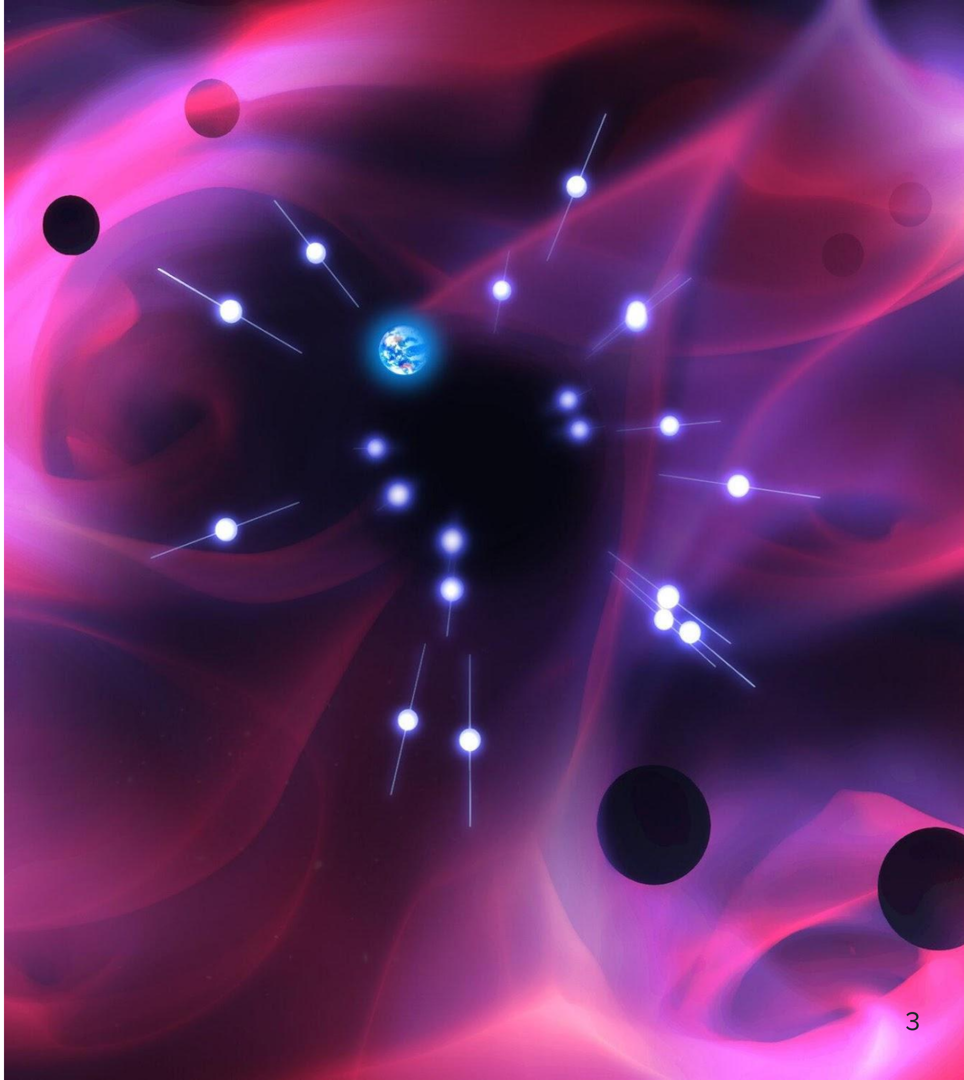
Hi from Spain!

Components of the lecture

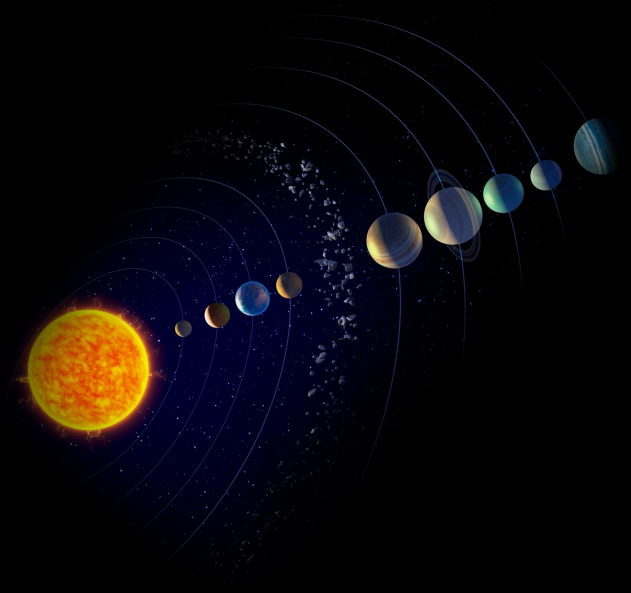
1. Pulsar Timing Arrays (PTAs)
2. Gravitational waves from Supermassive Black Hole Binaries
3. How do the signals look like in PTA data?
4. Realistic signals: towards the merger
5. Many SMBHBs



Pulsar Timing Arrays (PTAs): a recap

From pulsars to Pulsar Timing Arrays

Solar system and
the observatory



Interstellar medium (ISM)

- Dispersion measure (DM)
- Linear and quadratic DM: DM1, DM2
- ISM scintillation
- ISM scattering

* - not for gamma-ray data
** - Galactic centre challenges

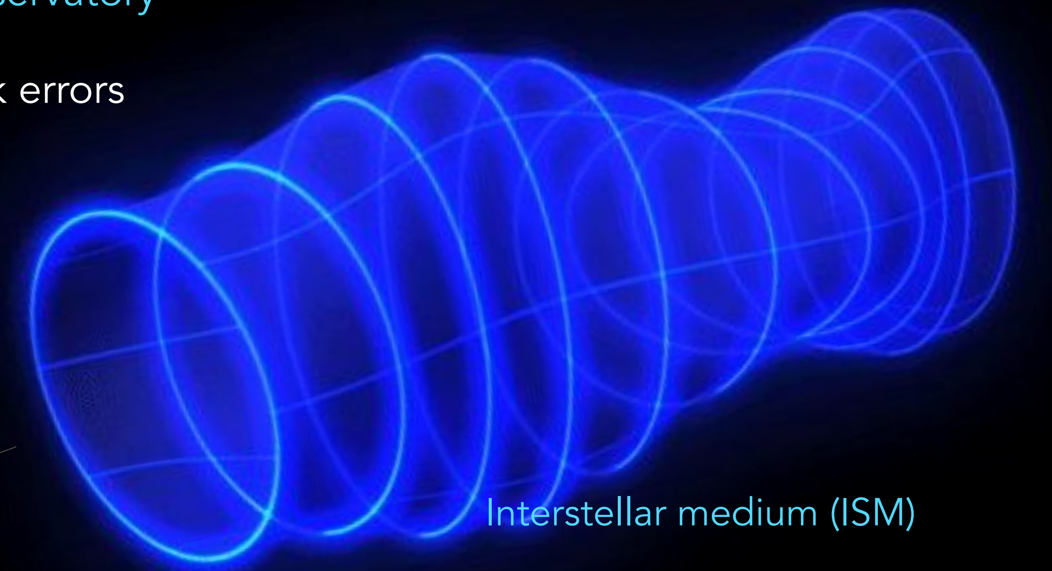
- Sky position, proper motion, parallax, etc.
- Spin frequency, derivatives: F0, F1, …
  (or spin period: P0, P1, …)
- Binary parameters
  - Relativistic, e.g. Shapiro delay, Lense-Thirring

Pulsar Timing Arrays and Stochastic Signals

Solar system and the observatory

- Clock errors

GRAVITATIONAL WAVES



Credit: ESA

Interstellar medium (ISM)

Dispersion measure (DM) variations

Pulsar

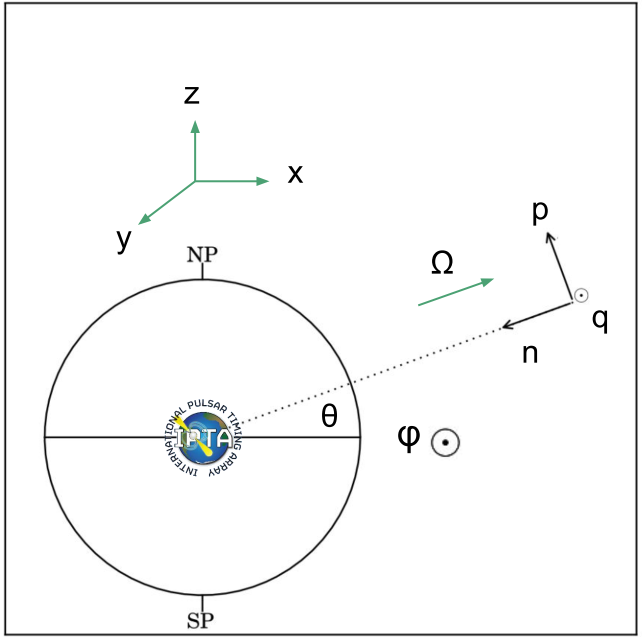
- Irregularities in pulsar rotation: spin noise

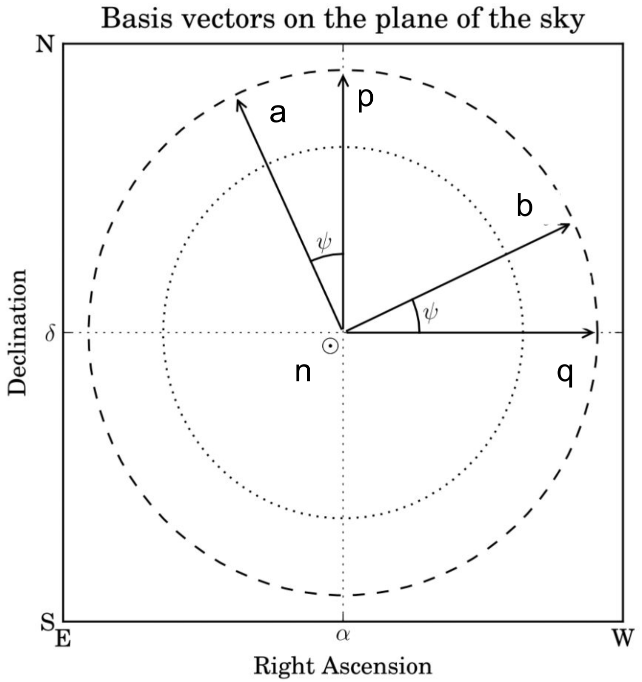
Stochastic nature, deterministic models:

- Glitches (predominantly young pulsars)
- Sudden changes in pulse profiles

Gravitational waves from Supermassive Black Hole Binaries

Gravitational wave polarizations





Adapted from: (1) The Geometry of Gravitational Wave Detection John T. Whelan School of Gravitational Waves, Warsaw, 2013 July 4, (2) arXiv: 2508.20007

Gravitational wave polarizations

Let us start by considering a plane wave traveling from an SMBHB to the Solar System barycenter (SSB) to which pulse arrival times are referenced. A unit vector that points from the CGW source to the SSB is

$$\hat{\Omega} = (\phi, \theta) = -\sin\theta\cos\phi\hat{x} - \sin\theta\sin\phi\hat{y} - \cos\theta\hat{z}, \tag{A1}$$

where $(\hat{x}, \hat{y}, \hat{z})$ is the Cartesian coordinate basis with the North Celestial Pole in the $\hat{z}$ direction, and the Vernal Equinox is in the $\hat{x}$ direction. A polar angle θ and an azimuthal angle ϕ of the source are related to the equatorial coordinate right ascension R.A. $= \phi$ and declination Dec. $= \pi/2 - \theta$.

The wave is characterized by the metric strain rank-2 tensor in the transverse traceless gauge, indexed with (a, b),

$$h_{ab}(t, \hat{\Omega}) = e^{+}_{ab}(\hat{\Omega})h_{+}(t, \hat{\Omega}) + e^{\times}_{ab}(\hat{\Omega})h_{\times}(t, \hat{\Omega}). \tag{A2}$$

Components $h_{+,\times}$ are the polarization amplitudes, and $e^{+,\times}_{ab}$ are the polarization tensors. Rotating the Cartesian coordinate system such that it is oriented towards a CGW source, we obtain new basis vectors in $(\hat{x}, \hat{y}, \hat{z})$ as in Arzoumanian et al. (2023):

$$\begin{aligned}
\hat{n} =&(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) = -\hat{\Omega}, \\
\hat{p} =&(\cos\psi\cos\theta\cos\phi - \sin\psi\sin\phi, \\
&\cos\psi\cos\theta\sin\phi + \sin\psi\cos\phi, -\cos\psi\sin\theta), \\
\hat{q} =&(\sin\psi\cos\theta\cos\phi + \cos\psi\sin\phi, \\
&\sin\psi\cos\theta\sin\phi - \cos\psi\cos\phi, -\sin\psi\sin\theta).
\end{aligned}$$

$$\boxed{\begin{aligned} e^{+}_{ab} &= \hat{p}_a\hat{p}_b - \hat{q}_a\hat{q}_b, \\ e^{\times}_{ab} &= \hat{p}_a\hat{q}_b + \hat{q}_a\hat{p}_b. \end{aligned}} \tag{A3}$$

Cardinal Tremblay, Goncharov, van Haasteren et al. (2025)

PTA antenna response

Cardinal Tremblay, Goncharov, van Haasteren et al. (2025)

The response of a PTA to the plane wave is described by the antenna pattern function (Romano & Cornish 2017)

$$\boxed{F^A(\hat{\Omega}) \equiv \frac{1}{2}\frac{\hat{u}^a\hat{u}^b}{1+\hat{\Omega}\cdot\hat{u}}e^A_{ab}(\hat{\Omega}),} \tag{A5}$$

where $\hat{u}$ is the unit vector pointing from the SSB to the pulsar. This function describes a pulsar's geometrical sensitivity to the CGW source (Taylor et al. 2016). Pulsar timing delays δt are then expressed as

$$\delta t(t,\hat{\Omega}) = F^+(\hat{\Omega})\delta t'_+(t) + F^\times(\hat{\Omega})\delta t'_\times(t), \tag{A6}$$

where we implied that $\delta t_{+,\times}$ is the time integral of $h_{+,\times}$, and $\delta t'_{+,\times}$ is the difference between the delay experienced at the SSB (Earth) and the signal induced at the pulsar. This difference is

$$\delta t'_{+,\times}(t) = \delta t_{+,\times}(t_\mathrm{p}) - \delta t_{+,\times}(t), \tag{A7}$$

where t is the time when the CGW passes the SSB and t_p is the time when the CGW passes the pulsar. One can then geometrically relate t and t_p,

$$t_p = t - \frac{L}{c}(1+\hat{\Omega}\cdot\hat{u}), \tag{A8}$$

where L as the distance to the pulsar.

Strain ➜ timing delays

$$h(t) = \begin{bmatrix} F_+ & F_\times \end{bmatrix} \begin{bmatrix} \cos 2\psi & -\sin 2\psi \\ \sin 2\psi & \cos 2\psi \end{bmatrix} \begin{bmatrix} h_+(t) \\ h_\times(t) \end{bmatrix}$$

$$\delta t(t) = \begin{bmatrix} F_+ & F_\times \end{bmatrix} \begin{bmatrix} \cos 2\psi & -\sin 2\psi \\ \sin 2\psi & \cos 2\psi \end{bmatrix} \begin{bmatrix} \delta t_+(t) \\ \delta t_\times(t) \end{bmatrix}$$

$$\delta t_{+,\times}(t) = \int_{t_0}^{t} h_{+\times}(t')dt'.$$

How do the signals look like in PTA data?

Continuous gravitational waves from SMBHBs

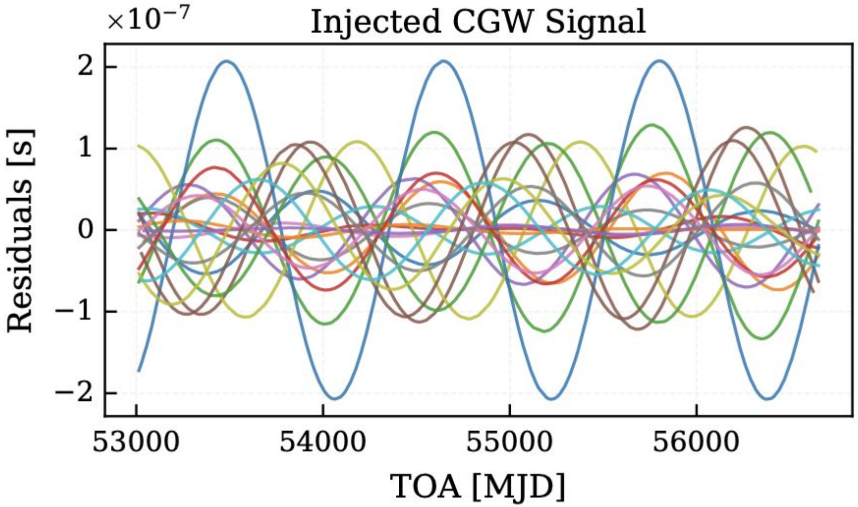


FIG. 2. Time-domain representation of the simulated data for a representative realization (Dataset A, $\rho_{\mathrm{tot}} \approx 30$). The top panel shows the total simulated timing residuals for all 20 pulsars, and the bottom panel shows the isolated CGW signal.

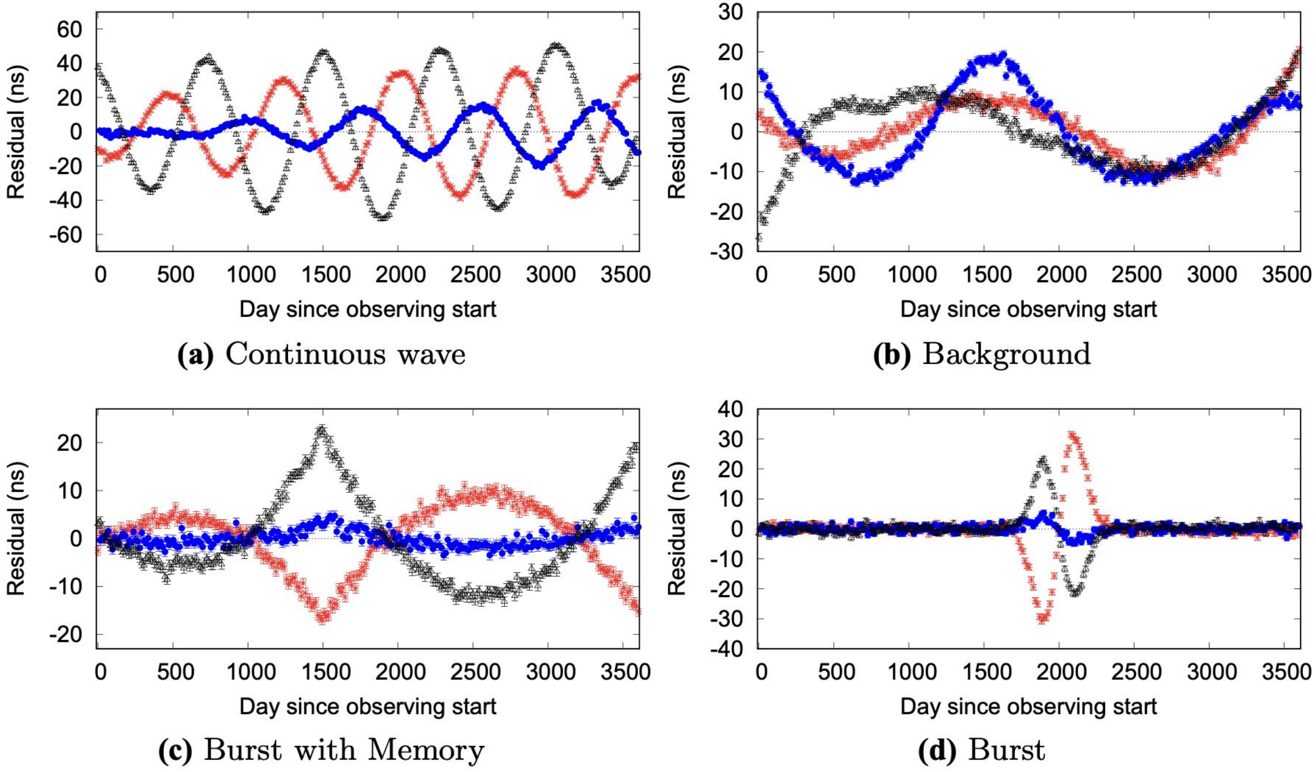
$$f_{\mathrm{GW}}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t - t_{\mathrm{c}})$$

$$h_0 = \frac{2(G\mathcal{M})^{5/3}(\pi f_{\mathrm{GW}})^{2/3}}{D_{\mathrm{L}}c^4}$$

arXiv: 1608.01940

Xuan Tao et al, arXiv: 2606.07330

Gravitational wave signals with PTAs



Sarah Burke-Spolaor, Stephen R. Taylor, Maria Charisi, et al. The astrophysics of nanohertz gravitational waves. A&A Rev., 27(1):5, June 2019.

Realistic signals: towards the merger

Gravitational Wave Memory in Pulsar Timing Arrays

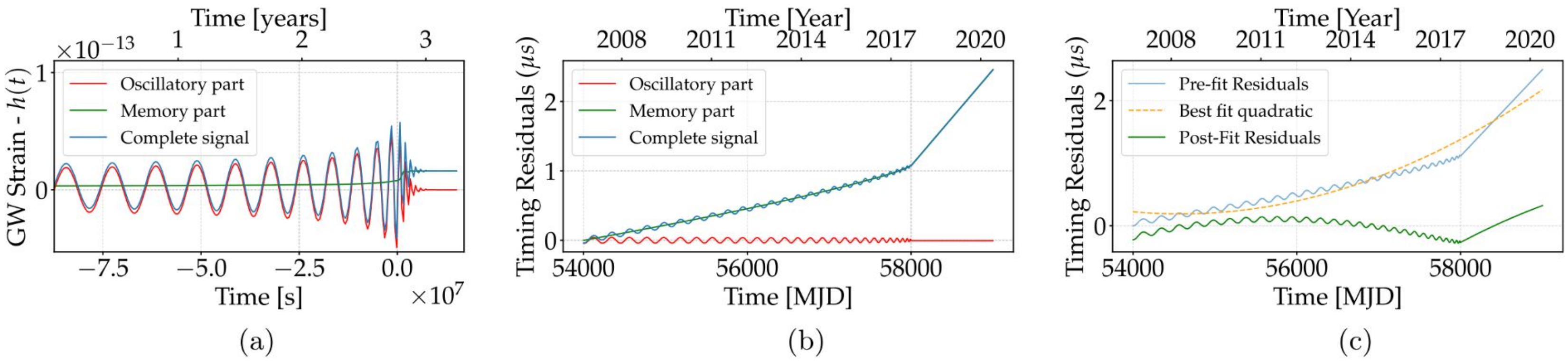
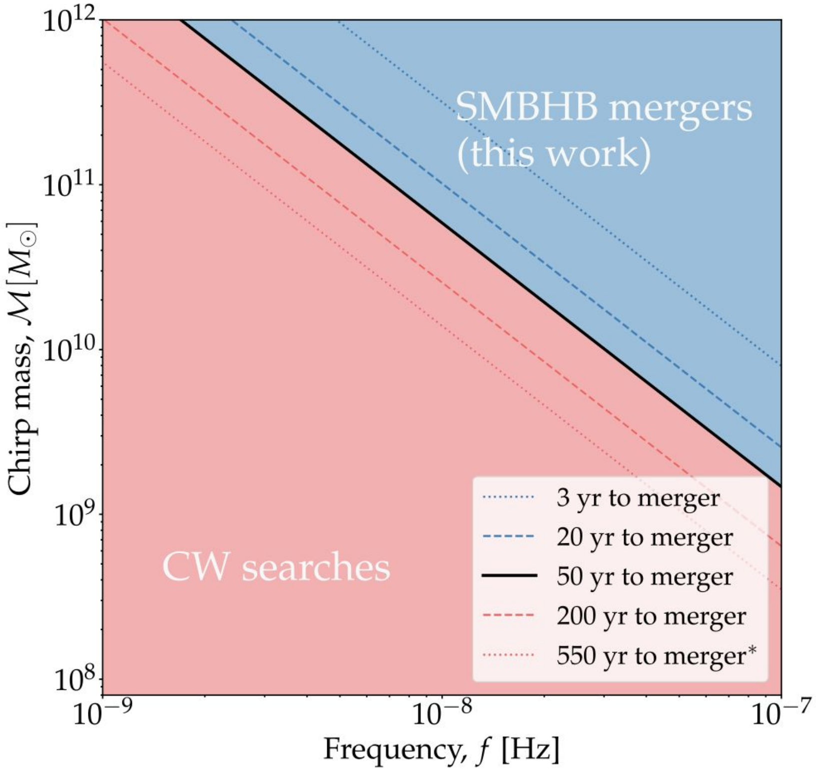


FIG. 1. Gravitational wave strain and timing residuals from a merger of a nonspinning SMBHB with parameters $\mathcal{M}_c = 10^{10} M_\odot$, $q = 1$, and $D_\mathrm{L} = 1000$ Mpc. (a) GW strain waveform $h(t)$ for three different combinations of radiative modes: $(l, m) = (2 \pm 2)$, $(2 \pm 1), (3 \pm 3)$ without memory (red), the null memory-only (2, 0) mode (green), and the complete signal including both oscillatory and memory contributions (blue). (b) The corresponding response induced by the GW source located at $(\mathrm{ra}, \mathrm{dec}) = (0°, 0°)$ on a pulsar at $(\mathrm{ra}, \mathrm{dec}) = (258.4564°, 7.7937°)$. Colors match those in (a), and the merger occurs at MJD 58000. (c) Postfit residuals after subtraction of the best-fit pulsar spin frequency and its derivatives. The full signal is shown in blue, the linear and quadratic spin-down model in orange, and the resulting postfit residuals in green.

Tomson, Goncharov, van Haasteren, “Finding supermassive black hole binary mergers in Pulsar Timing Array data” (2026)

Very rare signals, but hopefully we will see them with PTAs



Many SMBHBs

Bonus: deterministic ➔ stochastic

Strain polarizations and strain amplitude, h_0

$$\begin{cases} h_+(t) = h_0(1+\cos^2\iota)\cos\Phi(t), \\ h_\times(t) = -2h_0\cos\iota\sin\Phi(t), \end{cases}$$

RMS strain amplitude

$$h_{\rm rms}^2 = \langle h_+^2 + h_\times^2\rangle_{\Phi,\iota}$$

$$\langle\cos^2\iota\rangle|_{\mathcal{U}(\cos\iota)} = 1/3$$

$$\langle\cos^4\iota\rangle|_{\mathcal{U}(\cos\iota)} = 1/5$$

$$\langle\sin^2\Phi\rangle|_{\mathcal{U}(\Phi)} = 1/2$$

$$h_{\rm rms} = \frac{1}{2}\sqrt{\frac{32}{5}}h_0$$

$$\sqrt{\langle|\tilde{h}(f_0)|^2\rangle_{\mathcal{U}(\cos\iota)}} = \frac{T_{\rm obs}}{\sqrt{2}}h_{\rm rms}\Big|_{T_{\rm obs}\gg f_0^{-1}}$$

Fourier-domain strain

$$\tilde{h}(f) = \int h(t)e^{-2\pi f t}dt$$

Fourier-domain RMS strain

$$\sqrt{\langle|\tilde{h}(f_0)|^2\rangle} = \sqrt{\langle|\tilde{h}_+(f_0)|^2 + |\tilde{h}_\times(f_0)|^2\rangle_{\mathcal{U}(\cos\iota)}}.$$

Bonus: deterministic ➜ stochastic

Fourier-domain strain

$$\tilde{h}(f) = \int h(t) e^{-2\pi f t} dt$$

Fourier-domain RMS strain

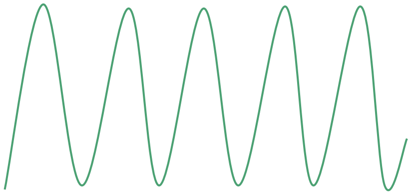
$$\sqrt{\langle|\tilde{h}(f_0)|^2\rangle} = \sqrt{\langle|\tilde{h}_+(f_0)|^2 + |\tilde{h}_\times(f_0)|^2\rangle}_{\mathcal{U}(\cos\iota)}.$$

$$\sqrt{\langle|\tilde{h}(f_0)|^2\rangle}_{\mathcal{U}(\cos\iota)} = \frac{T_{\rm obs}}{\sqrt{2}} h_{\rm rms}\Big|_{T_{\rm obs}\gg f_0^{-1}}$$

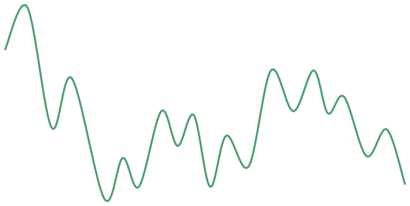
Strain Power Spectral Density, and characteristic strain h = √f S(f)

$$S(f) \equiv \lim_{T_{\rm obs}\to\infty} 2\frac{|\tilde{h}(f)|^2}{T_{\rm obs}}$$

$$h^2(f) \approx 2\frac{\langle|\tilde{h}(f_0)|^2\rangle}{T_{\rm obs}} f = \frac{2h_{\rm rms}^2 f}{T_{\rm obs}}\frac{T_{\rm obs}^2}{2} = h_{\rm rms}^2 f T_{\rm obs}$$



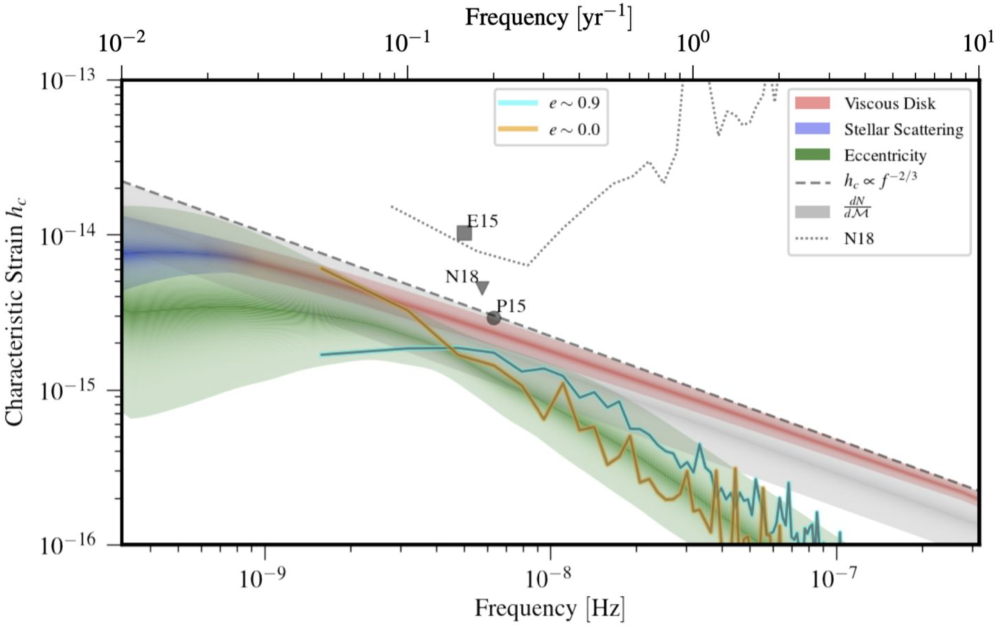
$$\boxed{h_0 = 2\frac{(G\mathcal{M})^{5/3}}{D_{\rm L}c^4}(\pi f)^{2/3} = 2h_{\rm cw}\sqrt{\frac{5}{32}\frac{\Delta f}{f}}}$$



Supermassive black hole binaries and PTAs

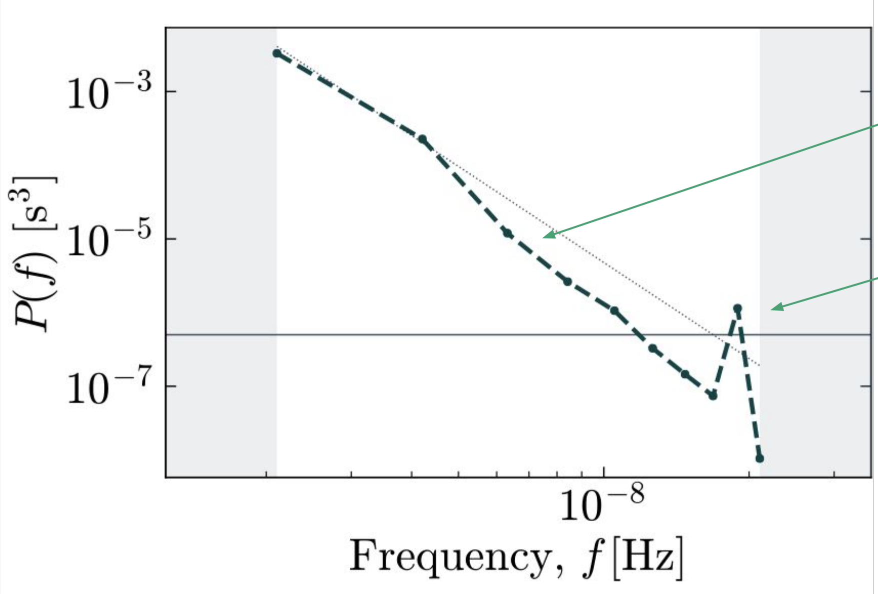
$$f_{\mathrm{GW}}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t - t_{\mathrm{c}})$$

$$h_0 = \frac{2(G\mathcal{M})^{5/3}(\pi f_{\mathrm{GW}})^{2/3}}{D_{\mathrm{L}}c^4}$$



Sarah Burke-Spolaor, Stephen R. Taylor, Maria Charisi, et al. The astrophysics of nanohertz gravitational waves. A&A Rev., 27(1):5, June 2019.

A joint analysis of deterministic and stochastic GWs



Gravitational Wave Background, model as h_c

Brightest source, individual SMBHB, model as h_c

$$h_0 = 2\frac{(G\mathcal{M})^{5/3}}{D_{\mathrm{L}}c^4}(\pi f)^{2/3} = 2h_{\mathrm{cw}}\sqrt{\frac{5}{32}\frac{\Delta f}{f}}$$

Goncharov, Sato-Polito, Bi, Zaldarriaga, arXiv 2606.18241

Thank you!